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## WILCOX FORMULA FOR VECTOR FIELDS ON BANACH MANIFOLDS

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We obtain an analogue of Wilcox–Snyder formula for flows of diffeomorphisms of  $C^m$ -smooth vector fields on infinite-dimensional Banach manifolds. For classical linear system this formula can be efficiently used, for example, to obtain Magnus expansion of solutions. The generalized Wilcox formula is obtained by using an extended Chronological Calculus for Banach manifold. We apply this formula to derive new structured differential equations which solutions approximate solutions of the original differential equation.

Keywords: flow of diffeomorphisms, Wilcox formula, chronological calculus, Magnus expansion.

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