CRITERIA OF MINIMAX SOLUTIONS FOR HAMILTON–JACOBI EQUATIONS WITH COINVARIANT FRACTIONAL-ORDER DERIVATIVES

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We consider Hamilton–Jacobi equations with coinvariant fractional-order derivatives, which arise in optimal control problems for dynamical systems whose evolution is described by differential equations with Caputo fractional derivatives. For upper, lower, and minimax (generalized) solutions of such equations, a number of criteria are established, which are expressed in terms of nonlocal conditions of stability with respect to characteristic differential inclusions satisfying a certain set of requirements and in the form of inequalities for suitably introduced derivatives of functionals in multivalued directions. In particular, these criteria make it possible to establish a correspondence between the results on the existence and uniqueness of minimax solutions of boundary value problems for the considered Hamilton–Jacobi equations obtained earlier under various assumptions.

Keywords: Hamilton–Jacobi equations, coinvariant derivatives, fractional-order derivatives, minimax solutions, characteristic differential inclusions, derivatives in multivalued directions.

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