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THE PROGRAM ITERATION METHOD AND THE RELAXATION PROBLEM

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The issues related to an approach–evasion differential game are considered: alternative solvability, construction of relaxations of an approach game problem, and construction of a solution based on the program iteration method. The case is considered when the set defining the phase constraints in a differential game with a closed target set may be nonclosed in the position space but has closed sections. For this situation, an alternative is established that is ideologically similar to the Krasovskii–Subbotin alternative under a certain correction of the classes of strategies. The question of constructing relaxations of the problem of approaching the target set in the presence of phase constraints is considered; it is assumed that the weakening of the conditions in terms of bringing the system to the target set and in terms of observing the phase constraints may be different, which is achieved by introducing a special priority coefficient. When a position of the game is fixed, the smallest size of a neighborhood of the target set is determined for which, with a proportional (in the sense of the mentioned coefficient) weakening of the phase constraints, the player interested in the approach can still guarantee it in an appropriate class of strategies (here, nonanticipation strategies or quasi-strategies). For the resulting main function of the position, a sequence of functions (positions) converging to this function is introduced based on a variant of the program iteration method operating in the space of sets with elements in the form of game positions. After that, a special operator on the function space (a program operator) is constructed, which implements this sequence by means of a "direct " iterative procedure and for which the main function itself is a fixed point. Thus, a new version of the program iteration method is implemented. A type of the quality functional with the following property is proposed: when a position is fixed, the value of the main function is the value of a game for the minimax–maximin of this functional.

Keywords: alternative, differential game, program iteration method, relaxation.

REFERENCES

1. Krasovskii N.N., Subbotin A.I. An alternative for the game problem of convergence. *J. Appl. Math. Mech.*, 1970, vol. 34, no. 6, pp. 948–965. doi: 10.1016/0021-8928(70)90158-9 .
2. Krasovskii N.N., Subbotin A.I. *Game-theoretical control problems*. NY: Springer, 1988, 517 p. ISBN: 978-1-4612-8318-8 . Original Russian text published in Krasovskii N.N., Subbotin A.I. *Pozitsionnye differentsial'nye igry*, Moscow: Nauka Publ., 1974, 456 p.
3. Isaacs R. *Differential games*. NY: John Wiley and Sons, 1965, 384 p. ISBN: 0471428604 . Translated to Russian under the title *Differentsial'nye igry*, Moscow: Mir Publ., 1967, 479 p.
4. Chentsov A.G. The program iteration method in a game problem of guidance. *Proc. Steklov Inst. Math.*, 2017, vol. 297, no. 1, pp. 43–61. doi: 10.1134/S0081543817050066 .
5. Chentsov A.G. Iterations of stability and the evasion problem with a constraint on the number of switchings of the formed control. *Izv. IMI UdGU*, 2017, vol. 49, pp. 17–54 (in Russian). doi: 10.20537/2226-3594-2017-49-02 .
6. Chentsov A.G. Some questions of differential game theory with phase constraints. *Izv. IMI UdGU*, 2020, vol. 56, pp. 138–184 (in Russian). doi: 10.35634/2226-3594-2020-56-10 .
7. Chentsov A.G. Relaxation of the game problem of guidance connected with alternative in guidance-evasion differential game. *Russian Universities Reports. Mathematics*, 2020, vol. 25, no. 130, pp. 196–244. doi: 10.20310/2686-9667-2020-25-130-196-244 . (in Russian)
8. Chentsov A.G., Khachay D.M. Relaxation of pursuit-evasion differential game and program absorption operator. *Vestn. Udmurt. Univ. Mat. Mekh. Komp. Nauki*, 2020, vol. 30, no. 1, pp. 64–91 (in Russian). doi: 10.35634/vm200106 .
9. Chentsov A.G., Khachay D.M. Program iterations method and relaxation of a pursuit-evasion differential game. In: Kondratenko Y., Chikrii A., Gubarev V., Kacprzyk J. (eds), *Advanced Control Techniques in*

Complex Engineering Systems: Theory and Applications, Ser. Studies in Systems, Decision and Control, vol. 203, Cham: Springer, 2019, pp. 129–161. doi: 10.1007/978-3-030-21927-7_7 .
10. Chentsov A.G., Khachay D.M. Relaxation of a dynamic game of guidance and program constructions of control. *Minimax Theory and its Applications*, 2020, vol. 5, no. 2, pp. 275–304.
11. Krasovskii N.N. *Igrovye zadachi o vstreche dvizhenii* [Game problems on the encounter of motions]. Moscow: Nauka Publ., 1970, 420 p.
12. Krasovskii N.N. A differential game of approach and evasion. I. *Engrg. Cybernetics*, 1973, vol. 11, no. 2, pp. 189–203.
13. Krasovskii N.N. A differential game of approach and evasion. II. *Engrg. Cybernetics*, 1973, vol. 11, no. 3, pp. 376–394.
14. Chentsov A.G. The structure of a certain game-theoretic approach problem. *Soviet Math. Dokl.*, 1975, vol. 16, no. 5, pp. 1404–1408.
15. Chentsov A. On a game problem of guidance. *Sov. Math., Dokl.*, 1976, vol. 17, pp. 73–77.
16. Chistyakov S.V. On solutions for game problems of pursuit. *Prikl. Mat. Mekh.*, 1977, vol. 41, no. 5, pp. 825–832 (in Russian).
17. Ukhobotov V.I. Construction of a stable bridge for a class of linear games. *J. Appl. Math. Mech.*, 1977, vol. 41, no. 2, pp. 350–354. doi: 10.1016/0021-8928(77)90021-1 .
18. Subbotin A.I. *Minimaksnye neravenstva i uravneniya Gamil'tona–Yakobi* [Minimax Inequalities and Hamilton–Jacobi Equations]. Moscow: Nauka Publ., 1991, 216 p. ISBN: 5-02-000139-2 .
19. Subbotin A.I. *Generalized solutions of first order PDEs: The dynamical optimization perspective*. Basel: Birkhäuser, 1995, 314 p. doi: 10.1007/978-1-4612-0847-1 .
20. Subbotin A.I. Continuous and discontinuous solutions of boundary value problems for first-order partial differential equations. *Russ. Acad. Sci., Dokl., Math.*, 1992, vol. 45, no. 2, pp. 257–261.
21. Subbotin A.I., Chentsov A.G. An iteration procedure for constructing minimax and viscous solutions to Hamilton–Jacobi equations. *Dokl. Math.*, 1996, vol. 53, no. 3, pp. 416–419.
22. Subbotin A.I., Chentsov A.G. An iterative procedure for constructing minimax and viscosity solutions to the Hamilton–Jacobi equations and its generalization. *Proc. Steklov Inst. Math.*, 1999, vol. 224, pp. 286–309.
23. Kuratowski K., Mostowski A. *Set theory*. Warszawa: PWN - Polish Scientific Publ., 1968, 417 p. ISBN: 9780444534170 . Translated to Russian under the title *Teoriya mnozhestv*. Moscow: Mir Publ., 1970, 416 p.
24. Dieudonné J. *Foundations of modern analysis*. NY: Acad. Press, 1960, 361 p. Translated to Russian under the title *Osnovy sovremennogo analiza*. Moscow: Mir Publ., 1964, 430 p.
25. Billingsley P. *Convergence of probability measures*. NY: Wiley, 1968, 253 p. ISBN: 0471072427. Translated to Russian under the title *Skhodimost' veroyatnostnykh mer*. Moscow: Nauka Publ., 1977, 352 p.
26. Dunford N., Schwartz J.T. *Linear Operators. I. General Theory.* NY: Interscience Publ., 1958, 858 p. ISBN: 0470226056 . Translated to Russian under the title *Lineinye operatory. Obshchaya teoriya.* Moscow: Inostr. Lit. Publ., 1962, 896 p.
27. Kryazhimskii A.V. On the theory of positional differential games of approach-evasion. *Soviet Math. Dokl.*, 1978, vol. 19, no. 2, pp. 408–412.
28. Subbotin A.I., Chentsov A.G. *Optimizatsiya garantii v zadachakh upravleniya* [Guarantee optimization in control problems]. Moscow: Nauka Publ., 1981, 288 p.
29. Chentsov A.G. *Metod programmnykh iteratsii dlya differentsial'noi igry sblizheniya-ukloneniya* [The method of program iterations for a differential approach-evasion game]. Sverdlovsk, 1979. Available from VINITI, no. 1933-79, 102 p.
30. Chentsov A.G. *Elementy konechno-additivnoi teorii mery*, I [Elements of Finitely Additive Measure Theory, I]. Yekaterinburg: USTU-UPI Publ., 2008, 388 p. ISBN: 978-5-321-01408-0 .

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