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ON THE CONSTRUCTION OF A DISCONTINUOUS PIECEWISE AFFINE SYNTHESIS IN A TARGET CONTROL PROBLEM

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A new method is proposed for the approximate solution of problems of solvability and control synthesis for a nonlinear system of ordinary differential equations. The method is based on the piecewise linearization (hybridization) of equations and on the use of the dynamic programming approach and the comparison principle. The main idea is to construct piecewise affine value functions and a feedback control of a special form. Two cases are considered: when these functions are continuous and when they may have discontinuities on certain sets in the state space. In both cases, we obtain internal estimates for the solvability sets of the original nonlinear system and a feedback control that takes the system's state vector to the target set on a given finite time interval.

Keywords: nonlinear dynamics, control synthesis, dynamic programming, comparison principle, linearization, switched system, piecewise affine value function.

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