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## ON INTERNAL ESTIMATES OF REACHABLE SETS FOR CONTINUOUS–DISCRETE SYSTEMS WITH DISCRETE MEMORY

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A linear continuous–discrete functional differential system with discrete memory is considered. The aim of the control is given by means of a finite collection of linear functionals with prescribed target values. The state and control variables are constrained by a system of linear pointwise and integral inequalities. A detailed description is given for the main relations and algorithms, which make it possible to derive an internal (lower by inclusion) estimate for the set of all target values attainable on the trajectories of the system under given constraints. The key constructions are based on the systematic use of the Cauchy operator of the continuous–discrete system. Various typical constraints are described and reduced to a unified form. Piecewise constant program controls are used in the construction of lower (by inclusion) estimates for the set of attainable values of objective functionals. The proposed estimates result from the solution of a special series of linear programs. The program controls that realize the conditional extremal target values are described.

Keywords: linear systems with aftereffect, hybrid systems, control problems, reachable sets.

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