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**ON THE ANALYTICAL CONSTRUCTION OF SOLUTIONS FOR ONE CLASS
OF TIME-OPTIMAL CONTROL PROBLEMS
WITH NONCONVEX TARGET SET****P. D. Lebedev, A. A. Uspenskii**

A time-optimal control problem with a circular velocity vectogram is considered. For one class of nonconvex planar target sets such that a part of their boundary coincides with a line segment, conditions are found that allow one to construct branches of singular (scattering) curves in analytical form. Explicit formulas are obtained for pseudovertrices, i.e., singular points of the boundary of the target set generating branches of the singular set. An analytical relation is revealed between the endpoints of different optimal trajectories that have the same initial conditions on the singular set and hit the target set in a neighborhood of a pseudoververtex. Formulas are found for the extreme points of branches of the singular set. The developed approaches to the exact construction of nonsmooth solutions of dynamic control problems are illustrated with examples.

Keywords: scattering curve, pseudoververtex, mapping, curvature.

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