AN EXISTENCE THEOREM AND AN APPROXIMATE SOLUTION METHOD FOR THE PFAFF EQUATION WITH CONTINUOUS COEFFICIENTS

A. A. Azamov, A. O. Begaliev

Pfaff equations with continuous coefficients are considered. A specific Cauchy problem for the Pfaff equation is transformed to an equivalent system of integral equations of a special type, which is overdetermined. It is shown that in the case of smooth coefficients the consistency of the system is equivalent to the Frobenius integrability criterion. A theorem on the existence of a solution for the obtained type of integral equations is presented. The solution is found by the Euler polygonal method, which allows one to construct an approximate solution of the Pfaff equation. An analog of Nagumo's theorem on the uniqueness of the solution to the Cauchy problem is also given.

Keywords: Pfaff equation, integral equation, consistency of a system, Frobenius criterion, existence theorem, Euler's broken lines, uniqueness of solution, Nagumo condition.

REFERENCES

1. Cartan É. Sur centaines expressions differentielles et le probleme de Pfaff. *Ann. Sci. de lÉ.N.S.*, 1899, vol. 16, pp. 239–332.

2. Han C.K. Pfaffian systems of Frobenius type and solvability of generic overdetermined PDE systems. In: Eastwood M., Miller W. (eds), *Symmetries and Overdetermined Systems of Partial Differential Equations*, Ser. The IMA Volumes Math. Appl., vol. 144, pp. 421–429.
   doi: 10.1007/978-0-387-73831-4 _21 .

3. Jouanolou J.P. *Equations de Pfaff algebriques.* Ser. Lecture Notes Math., vol. 708. Berlin: Springer, 1979, 255 p. doi: 10.1007/BFb0063393 .

4. Howard R. *Methods of thermodynamics.* NY: Blaisdell Publ. Comp., 1965, 233 p. ISBN: 0486694453 .

5. Godbillon C. *Géométrie différentielle et mécanique analytique.* Paris: Hermann, 2018, 184 p. ISBN: 978-2705656584 . Translated to Russian under the title *Differentsial'naya Geometriya i Analiticheskaya Mekhanika*, Moscow: Mir Publ., 1973, 188 p.

6. Rashevskii P.K. *Geometricheskaya teoriya uravnenii s chastnymi proizvodnymi* [Geometric theory of equations with partial derivatives]. Moscow; Leningrad: Gostekhizdat, 1947, 354 p.

7. Schutz B.F. *Geometrical methods of mathematical physics.* Cambridge; NY: Cambridge Univ. Press, 1980, 250 p. ISBN: 0521232716 . Translated to Russian under the title *Geometricheskie metody matematicheskoi fiziki*, Moscow: Mir Publ., 1984, 304 p.

8. Cartan H. *Formes differentielles.* Paris: Hermann, 1967, 186 p. ISBN: 978-2705667177 . Cartan H. *Calcul differentiel.* Paris: Hermann, 1967, 178 p. Translated to Russian under the title *Differentsial'noe ischislenie. Differentsial'nye formy.* Moscow: Mir Publ., 1971, 392 p.

9. Hartman P. *Ordinary differential equations.* Boston: Birkhäuser, 1982, 612 p. ISBN: 3764330686 . Translated to Russian under the title *Obyknovennye differentsial'nye uravneniya.* Moscow: Mir Publ., 1970, 720 p.

10. Tamura I. *Topologiya sloenii* [Topology of foliations]. Moscow: Mir Publ., 1979, 320 p.

11. Bedford E., Kalka M. Foliations and complex Monge–Ampere equations. *Comm. On Pure and Appl. Math.*, 1991, vol. 30, pp. 543–571. doi: 10.1002/cpa.3160300503 .

12. Luzatto S., Türeli S., War K. A Frobenius theorem for corank-1 continuous distributions in dimensions two and three. Available on *arXiv*: 1411.5896v5 [math.DG] 11 Apr 2016. 29 p.

13. Popescu P., Popescu M. Some aspects concerning the dynamics given by Pfaff forms. *Physics AUC*, 2011, vol. 21, pp. 195–202.

14. Hakopian H.A., Tonoyan M.G. Partial differential analogs of ordinary differential equations and systems. *New York J. Math.*, 2004, vol. 10, pp. 89–116.

15. Vasilevich N.D., Prokhorovich T.N. A linear Pfaff system of three equations on CP^m. *Differ. Equ.*, 2003, vol. 39, no. 6, pp. 896–898. doi: 10.1023/B:DIEQ.0000008417.90581.a4 .

16. Brunella M., Gustavo M.L. Bounding the degree of solutions to Pfaff equations. *Publ. Mat., Barc.*, 2000, vol. 44, no. 2, pp. 593–604. doi: 10.5565/PUBLMAT_44200_10 .

17. Cerveau D., Lins-Neto A. Holomorphic foliations in $CP(2)$ having an invariant algebraic curve. *Ann. Inst. Fourier*, 1991, vol. 41, no. 4, pp. 883–903. doi: 10.5802/aif.1278 .

18. Coutinho S. A constructive proof of the density of algebraic Pfaff equations without algebraic solutions. *Ann. Inst. Fourier*, 2007, vol. 57, no. 5, pp. 1611–1621. doi: 10.5802/aif.2308 .

19. Mendes L.G. Bounding the degree of solutions to Pfaff equations. *Publ. Mat., Barc.*, 2000, vol. 44, no. 2, pp. 593–604. doi: 10.5565/PUBLMAT_44200_10 .

20. Żoladek H. On algebraic solutions of algebraic Pfaff equations. *Studia Math.*, 1995, vol. 114, no. 2, pp. 117–126. doi: 10.4064/sm-114-2-117-126 .

21. Izobov N.A. On the existence of linear Pfaffian systems whose set of lower characteristic vectors has a positive plane measure. *Differ. Equ.*, 1997, vol. 33, no. 12, pp. 1626–1632.

22. Izobov N.A., Platonov A.S. Construction of a linear Pfaff equation with arbitrarily given characteristics and lower characteristic sets. *Differ. Equ.*, 1998, vol. 34, no. 12, pp. 1600–1607.

23. Spichekova N.V. On the behavior of integral surfaces of a Pfaff equation with a nonclosed singular curve. *Differ. Equ.*, 2005, vol. 41, no. 10, pp. 1509–1513. doi: 10.1007/s10625-005-0308-x .

24. Lefschetz S. *Differential equations: geometric theory.* N Y: Interscience Publishers, 1963, 390 p. Translated to Russian under the title *Geometricheskaya teoriya differentsial'nykh uravnenii.* Moscow: Izdatel'stvo inostrannoi literatury, 1961, 388 p.

25. Azamov A., Suvanov Sh., Tilavov A. Studying of behavior at infinity of vector fields on Poincare's sphere: revisited. *Qual. Theory Dyn. Syst.*, 2016, vol. 15, no. 1, pp. 211–220. doi: 10.1007/s12346-015-0176-6 .

26. Dryuma V. On geometrical properties of the spaces defined by the Pfaff equations. *Bul. Acad. Ştiinţe Repub. Mold., Mat.*, 2005, vol. 2005, no. 1(47), pp. 69–84.

27. Musen P. *On the application of Pfaff's method in the theory of variation of astronomical constants*, NASA Technical Note D-2301, Goddard Space Flight Center, Greenbelt, MD. 1964.

28. Mardare S. On Pfaff systems with L^p coefficients in dimension two. *Comptes Rendus Mathematique*, 2005, vol. 340, no. 12, pp. 879–884. doi: 10.1016/j.crma.2005.05.013 .

29. Mardare S. On Pfaff systems with L^p coefficients and their applications in differential geometry. *J. Math. Pures Appl.*, 2005, vol. 84, no. 12, pp. 1659–1692. doi: 10.1016/j.matpur.2005.08.002 .

30. Azamov A.A., Begaliyev A.O. Existence and uniqueness of the solution of a Cauchy problem for the Pfaff equation with continuous coefficients. *Uzb. Math. Journal*, 2019, no. 2, pp. 18–26. doi: 10.29229/uzmj.2019-2-2 .

31. Gaishun I.V. *Vpolne razreshimye mnogomernye differentsial'nye uravneniya* [Completely Solvable Multidimensional Differential Equations]. M.: Editorial URSS, 2004. 272 p.

32. Siu Y.T. *Partial differential equations with compatibility condition.* Available on https://www.coursehero.com/file/8864495/Lecture-notes-1/+.

33. Arutyunov A.V. *Lektsii po vypuklomu i mnogoznachnomu analizu* [Lectures on convex and set-valued analysis]. Moscow: Fizmatlit Publ., 2014, 184 p. ISBN: 978-5-9221-1558-2 .

34. Edwards R.E. *Functional analysis: theory and applications.* NY: Holt, Rhinehart and Winston, 1965, 783 p. ISBN: 0030505356 .

35. Coddington E., Levinson N. *Theory of ordinary differential equations.* NY: McGraw-Hill, 1955, 429 p. ISBN: 0070115427 . Translated to Russian under the title *Teoriya obyknovennykh differentsial'nykh uravnenii.* Moscow: Inostrannaya Literatura Publ., 1958, 475 p.

36. Cid J.Á On uniqueness criteria for systems of ordinary differential equations. *J. Math. Anal. Appl.*, 2003, vol. 281, no. 1, pp. 264–275. doi: 10.1016/S0022-247X(03)00096-9 .

37. Mejstrik T. Some remarks on Nagumo's theorem. *Czech. Math. J.*, 2012, vol. 62, no. 1, pp. 235–242. doi: 10.1007/s10587-012-0008-7 .

Abdulla Azamov, Academician of AS RUz, Prof., V.I.Romanovskii Institute of Mathematics of the Uzbekistan Academy of Sciences, Tashkent, 100174 Uzbekistan, e-mail: abdulla.azamov@gmail.com .

Aziz Begaliyev, PhD student, V.I.Romanovskii Institute of Mathematics of the Uzbekistan Academy of Sciences, Tashkent, 100174 Uzbekistan, e-mail: azizuzmu@mail.ru .