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ON THE POLYHEDRAL METHOD OF CONTROL SYNTHESIS IN THE PROBLEM OF TARGET EVASION IN DISCRETE-TIME SYSTEMS

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A conflict-control problem is considered for a linear discrete-time system with two controls, where the aim of the first control is to steer the trajectory of the system to a given target set, whereas the aim of the second control is opposite. Two subproblems arise here, namely, an approach problem and an evasion problem. It is assumed that the target set is a nondegenerate parallelepiped and both controls are subject to given parallelotope-valued constraints. The paper is devoted to the development of a fast polyhedral method of control synthesis in the evasion problem based on the construction of parallelotope-valued tubes. Two construction schemes for such tubes and the corresponding control strategies of avoiding the target set are studied. It is proved that under certain conditions both schemes provide particular solutions to the target evasion problem. The conditions imposed here are somewhat weaker than previously announced. Moreover, for both cases, guaranteed lower bounds are found for the deviation of the trajectory from the tube cross-sections. Here the last cross-section contains the target set by construction. The local properties of the schemes are compared.

Keywords: control system, systems with uncertainties, evasion problem, polyhedral methods, parallelotopes.

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