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OPTIMAL STATES OF DISTRIBUTED EXPLOITED POPULATIONS WITH PERIODIC IMPULSE SELECTION

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The dynamics of a population distributed on a torus is described by an equation of the Kolmogorov–Petrovsky–Piskunov–Fisher type in the divergence form. The population is exploited by periodic sampling of a constant distributed measurable ratio of its density. We prove that there exists a sampling ratio maximizing the time-averaged income in kind, i.e., a ratio that provides an optimal stationary exploitation in the long run.

Keywords: distributed population, Kolmogorov–Petrovsky–Piskunov–Fisher equation, impulse control, optimal solution.

MSC: 49J15

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