TRUDY INSTITUTA MATEMATIKI I MEKHANIKI UrO RAN

Vol. 27 No. 2 **2021**

STABLE BOUNDARY CONTROL OF A PARABOLIC EQUATION

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A problem of boundary control is considered for a differential equation with distributed parameters. It is required to design an algorithm that forms a feedback control and guarantees a prescribed quality of the controlled process. More exactly, the solution of this equation should track the solution of another equation, which is subject to an unknown perturbation. Methods for solving problems of this type for systems described by ordinary differential equations are well known and are presented, in particular, within the theory of positional control. In the present paper, we study a tracking problem in which the role of the control object is played by an equation with distributed parameters. It is assumed that the solutions of the equations are measured with an error, and the only available information about the perturbation is that it is an element of the space of functions summable with the square of the Euclidean norm; i.e., the perturbation can be unbounded. Taking into account these features of the problem, we design solution algorithms that are stable under information disturbances and computational errors. The algorithms are based on a combination of elements of the theory of ill-posed problems with the extremal shift method known in the theory of positional differential games.

Keywords: systems with distributed parameters, control.

MSC: 93B52, 93C20

DOI: 10.21538/0134-4889-2021-27-2-7-18

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Received January 31, 2021
Revised February 10, 2021
Accepted February 15, 2021

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Cite this article as: H. Akca, V. I. Maksimov. Stable boundary control of a parabolic equation, *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2021, vol. 27, no. 2, pp. 7–18 .