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NECESSARY OPTIMALITY CONDITIONS FOR SWITCHING SYSTEMS

A. S. Bortakovskii

We consider an optimal control problem for a switching system whose state vector contains both continuous and discretely varying components. The continuous and the discrete parts of the system are described by differential and recursive equations, respectively. The discrete part switches the operating modes of the continuous part and is itself influenced by the latter. The switching times and their number are not predefined. They are found as a result of optimizing the quality functional of the control process; here processes with instantaneous multiple switchings are not excluded. In deriving the necessary optimality conditions, we use small variations of the controls of the discrete part and variations of the switching times, which are represented by needle variations of the control of the continuous part of the system. The obtained conditions differ from the traditional equations for auxiliary variables due to the presence of instantaneous multiple switchings. The application of the optimality conditions is demonstrated by an academic example.

Keywords: hybrid systems, switching systems, optimal control.

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Alexandr Sergeevich Bortakovskii, Dr. Phys.-Math. Sci., Prof., Moscow Aviation Institute (National Research University), Moscow, 125993 Russia; National University of Science and Technology MISIS, Moscow, 119049 Russia, e-mail: asbortakov@mail.ru .

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