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MAXIMUM PRINCIPLE FOR AN OPTIMAL CONTROL PROBLEM WITH AN ASYMPTOTIC ENDPOINT CONSTRAINT

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Under conditions characterizing the dominance of the discounting factor, a complete version of the Pontryagin maximum principle for an optimal control problem with infinite time horizon and a special asymptotic endpoint constraint is developed. Problems of this type arise in mathematical economics in the studies of growth models.

Keywords: optimal control, infinite horizon, Pontryagin maximum principle, asymptotic endpoint constraint, growth models, sustainable development.

MSC: 49K15, 91B62

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