

ON THE RELAXATION OF A GAME PROBLEM OF APPROACH WITH PRIORITY ELEMENTS

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The issues related to the relaxation of a game problem of approach on a finite time interval are considered. In the original problem, it is assumed that the following sets are given: a target set closed in the position space and a set that determines state constraints and whose sections corresponding to fixed times are closed in the state space. The game termination conditions are relaxed by replacing these sets with their neighborhoods defined in different topologies of the position space; the “sizes” of the neighborhoods are related by a proportionality coefficient in the form of a priority parameter. For each value of this parameter and a fixed position, we find the value of the relaxed problem, which coincides with the minimax in the class of quasi-strategies for a special quality functional. It is established that the resulting position function depends on the parameter continuously as a mapping of the positive semiaxis to the Tikhonov power of the real line with the position space as the index set. Regions of uniform continuity are specified for the corresponding calculation functions (for a fixed position).

Keywords: differential game, quasi-strategy, program iteration method.

MSC:49J15, 49K15, 93C15, 49N70

DOI: 10.21538/0134-4889-2021-27-2-281-297

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Received January 18, 2021
 Revised January 29, 2021
 Accepted February 1, 2021

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Cite this article as: A. G. Chentsov. On the relaxation of a game problem of approach with priority elements, *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2021, vol. 27, no. 2, pp. 281–297 .