LINEAR EQUATIONS WITH DISCRETELY DISTRIBUTED FRACTIONAL DERIVATIVE IN BANACH SPACES

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We study the unique solvability of linear equations in Banach spaces with discretely distributed Gerasimov–Caputo fractional derivative in terms of analytic resolving families of operators. Necessary and sufficient conditions for the existence of such a family of operators are obtained in terms of the resolvent of a closed operator from the right-hand side of the equation, and the properties of this family are studied. These results are used to prove the existence of a unique solution to the Cauchy problem for a linear equation of the corresponding class with inhomogeneity which is either continuous in the norm of the graph of the operator from the right-hand side of the equation or Hölderian. Based on the abstract results obtained, we investigate the unique solvability of initial–boundary value problems for a class of equations with discretely distributed fractional time derivative and with polynomials in an elliptic self-adjoint differential operator with respect to spatial variables.

Keywords: Gerasimov–Caputo fractional derivative, discretely distributed fractional derivative, Cauchy problem, resolving family of operators, initial–boundary value problem.

REFERENCES

1. Nakhushev A.M. Continuous differential equations and their difference analogues. *Dokl. Math.*, 1988, vol. 37, no. 3, pp. 729–732.

2. Caputo M. Mean fractional-order-derivatives. Differential equations and filters. *Ann. Univ. Ferrara*, 1995, vol. 41, no. 1, pp. 73–84. doi: 10.1007/BF02826009 .

3. Sokolov I.M., Chechkin A.V., Klafter J. Distributed-order fractional kinetics. *Acta Physica Polonica B*, 2004, vol. 35, pp. 1323–1341.

4. Diethelm K., Ford N., Freed A.D., Luchko Y. Algorithms for the fractional calculus: A selection of numerical methods. *Computer Methods in Applied Mechanics and Engineering*, 2005, vol. 194, no. 6–8, pp. 743–773. doi: 10.1016/j.cma.2004.06.006 .

5. Pskhu A.V. *Uravneniya v chastnykh proizvodnykh drobnogo poryadka* [Partial differential equations of fractional order]. Moscow: Nauka Publ., 2005, 199 p. ISBN 5-02-033721-8 .

6. Atanacković T.M., Oparnica L., Pilipović S. On a nonlinear distributed order fractional differential equation. *J. Math. Anal. Appl.*, 2007, vol. 328, no. 1, pp. 590–608. doi: 10.1016/j.jmaa.2006.05.038 .

7. Kochubei A.N. Distributed order calculus and equations of ultraslow diffusion. *J. Math. Anal. Appl.*, 2008, vol. 340, no. 1, pp. 252–281. doi: 10.1016/j.jmaa.2007.08.024 .

8. Fedorov V.E., Abdrakhmanova A.A. A class of initial value problems for distributed order equations with a bounded operator. *Stability, Control and Differential Games*. A. Tarasyev, V. Maksimov, T. Filippova (eds). Cham: Springer Nature, 2020, pp. 251–262. doi: 10.1007/978-3-030-42831-0_22 .

9. Fedorov V.E., Streletskaya E.M. Initial-value problems for linear distributed-order differential equations in Banach spaces. *Electronic J. Diff. Eq.*, 2018, vol. 2018, no. 176, pp. 1–17.

10. Fedorov V.E., Phuong T.D., Kien B.T., Boyko K.V., Izhberdeeva E.M. A class of distributed order semilinear equations in Banach spaces. *Chelyab. Fiz.-Mat. Zh.*, 2020, vol. 5, no. 3, pp. 342–351 (in Russian). doi: 10.47475/2500-0101-2020-15308 .

11. Fedorov V.E. On generation of an analytic in a sector resolving operators family for a distributed order equation. *Zap. Nauchn. Sem. POMI*, 2020, vol. 489, pp. 113–129 (in Russian).

12. Fedorov V.E. Generators of analytic resolving families for distributed order equations and perturbations. *Mathematics*, 2020, vol. 8, no. 1306, pp. 1–15. doi: 10.3390/math8081306 .

13. Pskhu A.V. Fractional diffusion equation with discretely distributed differentiation operator. *Sib. Elektron. Mat. Izv.*, 2016, vol. 13, pp. 1078–1098 (in Russian). doi: 10.17377/semi.2016.13.086 .

14. Kostić M. Degenerate multi-term fractional differential equations in locally convex spaces. *Publication de l'Institut Mathématique. Nouvelle série*, 2016, vol. 100, no. 114, pp. 49–75. doi: 10.2298/PIM1614049K .

15. Novozhenova O.G. Life and science of Alexey Gerasimov, one of the pioneers of fractional calculus in Soviet Union. *Fractional Calculus and Applied Analysis*, 2017, vol. 20, no. 3, pp. 790–809. doi: 10.1515/fca-2017-0040 .

16. Podlubny I. *Fractional differential equations.* San Diego; Boston: Acad. Press, 1999, 340 p. ISBN: 0-12-558840-2 .

17. Prüss J. *Evolutionary integral equations and applications.* Basel: Springer, 1993, 369 p. ISBN: 978-3-0348-8570-6 .

18. Arendt W., Batty C.J.K., Hieber M., Neubrander F. Vector-valued Laplace transforms and Cauchy problems. Basel: Springer, 2011, 539 p. ISBN: 978-3-0348-0087-7 .

19. Bajlekova E.G. *Fractional evolution equations in Banach spaces.* PhD thesis; Eindhoven: Eindhoven University of Technology: University Press Facilities, 2001. 117 p.

20. Yosida K. *Functional Analysis.* Berlin; Heidelberg: Springer-Verlag, 1965, 475 p. ISBN: 978-3-642-52814-9 . Translated to Russian under the title *Funktsional'nyi analiz.* Moscow: Mir Publ., 1967, 616 p.

21. Goldstein J.A. Semigroups and second-order differential equations. *Journal of Functional Analysis*, 1969, vol. 4, no. 1, pp. 50–70. doi: 10.1016/0022-1236(69)90021-4 .

22. Triebel H. *Interpolation theory, function spaces, differential operators.* Amsterdam: North-Holland Publ., 1978, 528 p. ISBN: 0720407109 . Translated to Russian under the title *Teoriya interpolyatsii. Funktsional'nye prostranstva. Differentsial'nye operatory.* Moscow: Mir Publ., 1980, 664 p.

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