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REGULARIZATION OF THE PONTRYAGIN MAXIMUM PRINCIPLE IN A CONVEX OPTIMAL BOUNDARY CONTROL PROBLEM FOR A PARABOLIC EQUATION WITH AN OPERATOR EQUALITY CONSTRAINT

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We consider the regularization of the classical optimality conditions — the Lagrange principle (LP) and the Pontryagin maximum principle (PMP) — in a convex optimal control problem for a parabolic equation with an operator equality constraint and with a boundary control. The set of admissible controls of the problem is traditionally embedded into the space of square-summable functions. However, the objective functional is not, generally speaking, strongly convex. The derivation of regularized LP and PMP is based on the use of two regularization parameters. One of them is "responsible" for the regularization of the dual problem, while the other is contained in a strongly convex regularizing addition to the objective functional of the original problem. The main purpose of the regularized LP and PMP is the stable generation of minimizing approximate solutions in the sense of J. Warga. The regularized LP and PMP are formulated as existence theorems in the original problem of minimizing approximate solutions consisting of minimals of its regular Lagrange function. They "overcome" the ill-posedness properties of the LP and PMP and are regularizing algorithms for solving the optimal control problem. Particular attention is paid to the proof of the PMP in the problem of minimizing the regular Lagrange function and obtaining on this basis the regularized PMP in the original optimal control problem as a consequence of the regularized LP.

Keywords: convex optimal control, parabolic equation, operator constraint, boundary control, minimizing sequence, regularizing algorithm, Lagrange principle, Pontryagin maximum principle, dual regularization.

MSC: 49K20, 49N15, 47A52

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