

**WEAK* APPROXIMATIONS FOR THE SOLUTION
OF A DYNAMIC RECONSTRUCTION PROBLEM****N. N. Subbotina, E. A. Krupennikov**

We consider the problem of the dynamic reconstruction of an observed state trajectory $x^*(\cdot)$ of an affine deterministic dynamic system and the control that has generated this trajectory. The reconstruction is based on current information about inaccurate discrete measurements of $x^*(\cdot)$. A correct statement of the problem on the construction of approximations $u^l(\cdot)$ of the normal control $u^*(\cdot)$ generating $x^*(\cdot)$ is refined. The solution of this problem obtained using the variational approach proposed by the authors is discussed. Conditions on the input data and matching conditions for the approximation parameters (parameters of the accuracy and frequency of measurements of the trajectory and an auxiliary regularizing parameter) are given. Under these conditions, the reconstructed trajectories $x^l(\cdot)$ of the dynamical system converge uniformly to the observed trajectory $x^*(\cdot)$ in the space of continuous functions C as $l \rightarrow \infty$. It is proved that the proposed controls $u^l(\cdot)$ converge weakly* to $u^*(\cdot)$ in the space of summable functions L^1 .

Keywords: dynamic reconstruction problems, convex–concave discrepancy, problems of calculus of variations, Hamiltonian systems.

MSC: 65K10, 34A55, 49K15

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