

MATRIX RESOLVING FUNCTIONS IN A LINEAR PROBLEM OF GROUP PURSUIT WITH MULTIPLE CAPTURE

N. N. Petrov

A problem of pursuit of one or several evaders by a group of pursuers is considered in a finite-dimensional Euclidean space. The problem is described by the system

$$\dot{z}_{ij} = A_{ij}z_{ij} + u_i - v_j, \quad u_i \in U_i, \quad v_j \in V_j.$$

The aim of the group of pursuers is to capture at least q evaders, where each evader must be captured by at least m different pursuers; the capture moments may be different. The terminal sets are the origin. Matrix resolving functions, which generalize scalar resolving functions, are used as a mathematical basis. Sufficient conditions for the multiple capture of one evader in the class of quasi-strategies are obtained. Under the assumption that the evaders use program strategies and each pursuer captures at most one evader, sufficient conditions for the solvability of the problem on the multiple capture of a given number of evaders are obtained in terms of the initial positions. Hall's theorem on a system of distinct representatives is used to prove the main theorem. Examples are given to illustrate the obtained results.

Keywords: differential game, pursuer, evader, group pursuit.

MSC: 49N79, 49N70, 91A24

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Nikolai Nikandrovich Petrov, Dr. Phys.-Math. Sci., Prof., Udmurt State University, Izhevsk, 426034 Russia, e-mail: kma3@list.ru.

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