

**LINEAR CONTROLLED OBJECTS WITH STATE CONSTRAINTS.
APPROXIMATE CALCULATION OF REACHABLE SETS****M. S. Nikol'skii**

Linear controlled objects are intensively studied in modern control theory. An important dynamic characteristic of such objects is their reachable sets. For example, these sets are used in optimal control theory to formulate problems that are interesting for applications. Knowing reachable sets at different times, one can roughly estimate the dynamic capabilities of the controlled object under study. Note that in the absence of state constraints, the techniques of support functions are effective for calculating these sets. Under state constraints, the calculation becomes more complicated. We develop a method for the approximate calculation of reachable sets for linear controlled objects under constraints. The convergence of these approximations to the desired reachable set in the sense of the Hausdorff metric is proved. It is assumed that the state constraint and the set constraining the control are convex and compact. To construct approximations, we use the Cauchy formula and a uniform partition of the interval $[0, T]$ on which the motion occurs. An estimate for the rate of convergence of approximations to the required set is obtained under some additional assumptions.

Keywords: linear controlled objects, phase constraints, reachable sets, Cauchy formula.

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