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ON THE SOLVABILITY OF THE PROBLEM OF SYNTHESIZING DISTRIBUTED AND BOUNDARY CONTROLS IN THE OPTIMIZATION OF OSCILLATION PROCESSES

A. Kerimbekov

We study the solvability of the problem of synthesis of distributed and boundary controls in the optimization of oscillation processes described by partial integro-differential equations with the Fredholm integral operator. Functions of external and boundary actions are nonlinear with respect to the controls. For the Bellman functional, an integro-differential equation of a specific form is obtained and the structure of its solution is found, which allows this equation to be represented as a system of two equations of a simpler form. An algorithm for constructing a solution to the problem of synthesizing distributed and boundary controls is described, and a procedure for finding the controls as a function (functional) of the state of the process is described.

Keywords: integro-differential equation, Fredholm operator, generalized solution, Bellman functional, Fréchet differential, optimal control synthesis.

MSC: 49K20

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Akylbek Kerimbekov, Dr. Phys.-Math. Sci., Prof., Kyrgyz-Russian Slavic university, Bishkek, 720022 Kyrgyzstan, e-mail: akl7@rambler.ru .

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