

## ON THE UNIQUENESS OF A SOLUTION TO THE PROBLEM OF FINDING A COMPOSITE SOURCE IN THE HEAT EQUATION

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An initial-boundary value problem is considered for a two-dimensional heat equation with a source. The source is composite; namely, it is the sum of two unknown functions of spatial variables multiplied by given power functions of time. An inverse problem is posed, which consists in determining the two unknown functions from additional information about the solution of the initial-boundary value problem, which is a function of time and of one of the spatial variables. It is shown that such an inverse problem has an infinite set of solutions in the general case. Theorems on the uniqueness of a solution of the inverse problem in some special classes of unknown functions are proved.

Keywords: heat equation, unknown source, inverse problem, uniqueness of solution.

MSC: 65M32

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