

ON THE EXISTENCE OF A PERIODIC SOLUTION OF THE LIÉNARD SYSTEM WITH IMPULSE EFFECT

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We consider a system of Liénard differential equations with impulse effect

$$\frac{dx}{dt} = z - F(x), \quad \frac{dz}{dt} = -g(x), \quad \text{for } x \neq 0,$$

$$\Delta x = 0, \quad \Delta z = J(z) \quad \text{for } x = 0.$$

Sufficient conditions for the existence of a periodic solution of this system are obtained.

Keywords: systems of differential equations with impulse effect, Liénard system, limit cycle.

REFERENCES

1. Bainov D.D., Simeonov P.S. *Systems with impulse effect: stability, theory and applications*. N Y; Chichester; Brisbane; Toronto: Halsted Press, 1989, 256 p. ISBN: 0-7458-0457-8.
2. Lakshmikantham V., Bainov D.D., Simeonov P.S. *Theory of impulsive differential equations*. Singapore; N J; London: World Scientific, 1989, 273 p. doi: 10.1142/0906.
3. D’Onofrio A. Pulse vaccination strategy in the SIR epidemic model: Global asymptotic stable eradication in presence of vaccine failures. *Mathematical and Computer Modelling*, 2002, vol. 36, pp. 473–489. doi: 10.1016/S0895-7177(02)00177-2.
4. Smith R.J., Wahl L.M. Distinct effects of protease and reverse transcriptase inhibition in an immunological model of HIV-1 infection with impulsive drug effects. *Bulletin Math. Biology*, 2004, vol. 66, pp. 1259–1283. doi: 10.1016/J.BULM.2003.12.004.
5. Jiao J., Liu Z., Li L., Nie X. Threshold dynamics of a stage-structured single population model with non-transient and transient impulsive effects. *Applied Math. Letters*, 2019, vol. 97, no. 1, pp. 88–92. doi: 10.1016/j.aml.2019.05.024.
6. Ignatyev A.O. Method of Lyapunov functions in problems of stability of systems of differential equations with impulse action. *Sb. Math.*, 2003, vol. 194, no. 10, pp. 1543–1558. doi: 10.1070/SM2003v194n10ABEH000776.
7. Dvirnyi A.I., Slyn’ko V.I. A counterpart of A.M. Molchanov’s critical case for impulse systems. *Autom. Remote Control*, 2015, vol. 76, no. 6, pp. 945–956. doi: 10.1134/S0005117915060016.
8. Haddad W.M., Chellaboina V., Nersesov S.G. *Impulsive and hybrid dynamical systems: stability, dissipativity, and control*. Princeton: Princeton University Press, 2006, 520 p. doi: 10.1515/9781400865246.
9. Graef J.R., Kadari H., Ouahab A., and Oumansour A. Existence results for systems of second-order impulsive differential equations. *Acta Math. Univ. Comenianae*, 2019, vol. 88, no. 1, pp. 51–66.
10. Levinson N. and Smith O. A general equation for relaxation oscillations. *Duke Math. J.*, 1942, vol. 9, no. 2, pp. 382–403. doi: 10.1215/S0012-7094-42-00928-1.
11. Ignat’ev A.O. Estimate for the amplitude of the limit cycle of the Liénard equation. *Diff. Equat.*, 2017, vol. 53, no. 3, pp. 302–310. doi: 10.1134/S0012266117030028.
12. Ignatyev A.O. The domain of existence of a limit cycle of Liénard system. *Lobachevskii J. Math.*, 2017, vol. 38, no. 2, pp. 271–279. doi: 10.1134/S199508021702010X.

13. Carletti T., Villari G. A note on existence and uniqueness of limit cycles for Liénard systems. *J. Math. Analysis Appl.*, 2005, vol. 307, no. 2, pp. 763–773. doi: 10.1016/j.jmaa.2005.01.054.
14. Sabatini M., Vilari G. On the uniqueness of limit cycles for Lienard equation: the legacy of G. Sansone. *Matematiche (Catania)*, 2010, vol. 65, no. 2, pp. 201–214.
15. Belley J.M., Guen R. Periodic van der Pol equation with state dependent impulses. *J. Math. Analysis Appl.*, 2015, vol. 426, pp. 995–1011. doi: 10.1016/j.jmaa.2015.02.026.
16. Herrera L., Montano O., Orlov Yu. Hopf bifurcation of hybrid Van der Pol oscillators. *Nonlinear Analysis: Hybrid Systems*, 2017, vol. 26, pp. 225–238. doi: 10.1016/j.nahs.2017.05.003.
17. Ding B., Pan S., Ding C. The index of impulsive periodic orbits. *Nonlinear Analysis*, 2020, vol. 192, pp. 1–9. doi: 10.1016/j.na.2019.111659.

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