

UDK 512.542

**ON THE INTERSECTIONS OF NILPOTENT SUBGROUPS IN FINITE GROUPS
WITH SOCLE $L_3(q)$ or $U_3(q)$**

V. I. Zenkov

MSC: 20D06, 20D15, 20D20, 20D30

DOI: 10.21538/0134-4889-2021-27-1-70-78

Earlier, the author described up to conjugation all pairs (A, B) of nilpotent subgroups A and B in a finite group G with socle $L_2(q)$ for which $A \cap B^g \neq 1$ for any element g of G . A similar description was obtained later by the author for primary subgroups A and B of a finite group G with socle $L_n(2^m)$. In this paper, we describe up to conjugation all pairs (A, B) of nilpotent subgroups A and B of a finite group G with socle $L_3(q)$ or $U_3(q)$ for which $A \cap B^g \neq 1$ for any element g of G . The obtained results confirm in the considered cases the hypothesis that for a finite simple non-Abelian group G and its nilpotent subgroup N there is an element $g \in G$ such that $N \cap N^g = 1$ (Problem 15.40 from “The Kourovka Notebook”).

Keywords: finite group, nilpotent subgroup, intersection of subgroups, Fitting subgroup.

REFERENCES

1. Isaacs I.M. *Finite group theory*. Providence, RI: AMS, 2008. 350 p.
2. Mazurov V.D., Zenkov V.I. The intersection of Sylow subgroups in finite groups. *Algebra and Logic*, 1996, vol. 35, no. 4, pp. 236–240.
3. Zenkov V.I. Intersections of nilpotent subgroups in finite groups. *Fund. i Prikl. Matematika*, 1994, vol. 56, pp. 1–91 (in Russian).
4. *Unsolved problems in group theory. The Kourovka notebook*. No. 19. Novosibirsk: Inst. Math. SO RAN Publ., 2018, 250 p. Available on: <http://math.nsc.ru/~alglog/19tkt.pdf>.
5. Kurmazov R.K. On the intersection of conjugate nilpotent subgroups in permutation groups. *Siberian Math. J.*, 2013, vol. 54, no. 1, pp. 73–77. doi: 10.1134/S0037446613010102.
6. Zenkov V.I. Intersections of two nilpotent subgroups in finite groups with socle $L_2(q)$. *Siberian Math. J.*, 2016, vol. 57, no. 6, pp. 1002–1010. doi: 10.1134/S0037446616060070.
7. Zenkov V.I. Intersections of primary subgroups in nonsoluble finite groups isomorphic to $L_n(2^m)$. *Siberian Math. J.*, 2018, vol. 59, no. 2, pp. 264–269. doi: 10.1134/S0037446618020088.
8. Gorenstein D., Lyons R., Solomon R. The classification of the finite simple groups, Number 3. Providence, RI: AMS, 1998. 420 p.
9. Gorenstein D., *Finite simple groups. An introduction to their classification*. N Y; London: Plenum Press, 1982, 352 p.
10. Zenkov V.I. On intersections of abelian and nilpotent Subgroups in finite groups. II, *Math. Notes*, 2019, vol. 105, no. 3, pp. 366–375. doi: 10.1134/S0001434619030076.
11. Zenkov V.I. On intersections of two nilpotent subgroups in small finite groups. *Sib. Elektron. Math. Izv.*, 2016, vol. 13, pp. 1099–1115 (in Russian).
12. Gorenstein D., Lyons R. The local structure of finite groups of characteristic 2 type. *Mem. Amer. Math. Soc.*, vol. 42, 1983, pp. 1–731. doi: 10.1090/memo/0276.
13. *Atlas of finite group* / Conway J.H. [et. al.]. Oxford: Clarendon Press, 1985, 252 p.
14. Aschbacher M., Seitz G. Involutions in Chevalley groups over fields of even order. *Nagoya Math. J.*, 1974, vol. 60, pp. 1–91.
15. Yang Y. Regular orbits of nilpotent subgroups of solvable linear groups. *J. Algebra*, 2011, vol. 325, no. 1, pp. 56–69.
16. Aschbacher M. Overgroups of Sylow subgroups in sporadic groups. *Mem. Amer. Math. Soc.*, 1986, vol. 60, no. 343. pp. 1–235. doi: 10.1090/memo/0343.

17. Gagen T. *Topics in finite groups*. Cambridge: Cambridge University Press, 1976, 83 p.

Received September 22, 2020

Revised December 20, 2020

Accepted January 11, 2021

Funding Agency: This work was supported by the Russian Foundation for Basic Research (project no. 20-01-00456) and by the Russian Academic Excellence Project (agreement no. 02.A03.21.0006 of August 27, 2013, between the Ministry of Education and Science of the Russian Federation and Ural Federal University).

Zenkov Victor Ivanovich, Dr. Phys.-Math. Sci., Krasovskii Institute of Mathematics and Mechanics Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620108 Russia; Ural Federal University, 620000 Russia, e-mail: vli9z52@mail.ru .

Cite this article as: V. I. Zenkov. On the intersections of nilpotent subgroups in finite groups with socle $L_3(q)$ or $U_3(q)$, *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2021, vol. 27, no. 1, pp. 70–78 .