

**GENERATING SETS OF CONJUGATE INVOLUTIONS OF THE GROUPS  $SL_n(q)$   
FOR  $n = 4, 5, 7, 8$  AND ODD  $q$**

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**MSC:** 20G40

**DOI:** 10.21538/0134-4889-2021-27-1-62-69

In 2009 J. M. Ward answered for sporadic and alternating groups and for projective special linear groups  $PSL_n(q)$  over a field of odd order  $q$  except for the case  $q = 9$  for  $n \geq 4$  and, for  $n = 6$ , the case  $q \equiv 3 \pmod{4}$  Question 14.69c from *The Kourovka Notebook* posed by the second author of the present paper: *For every finite simple nonabelian group  $G$ , find the minimum number  $n_c(G)$  of generating conjugate involutions whose product is 1.* It is known that  $n_c(G) \geq 5$  for any simple nonabelian group  $G$ . We discard the constraint  $q \neq 9$  for the dimensions  $n = 4, 5, 7, 8$ . It turns out that in these dimensions the generating quintuples of conjugate involutions with the product equal to 1 for special linear groups  $SL_n(q)$  and, consequently, for  $PSL_n(q)$ , specified by Ward, are also suitable for  $q = 9$ .

Keywords: spacial linear group over a finite field, generating triples of conjugate involutions.

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Received August 6, 2020

Revised September 20, 2020

Accepted January 11, 2021

**Funding Agency:** This work was supported by the Krasnoyarsk Mathematical Center, which is financed by the Ministry of Science and Higher Education of the Russian Federation within the project for the establishment and development of regional centers for mathematical research and

education (agreement no. 075-02-2020-1534/1), and by the Russian Foundation for Basic Research (project no. 19-01-00566).

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Cite this article as: I. Yu. Efimov, Ya. N. Nuzhin. Generating sets of conjugate involutions of the groups  $SL_n(q)$  for  $n = 4, 5, 7, 8$  and odd  $q$ , *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2021, vol. 27, no. 1, pp. 62–69.