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FINITE GROUPS WITH FOUR CONJUGACY CLASSES OF MAXIMAL SUBGROUPS. III

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We continue the study of finite groups with exactly four conjugacy classes of maximal subgroups. Groups with this property are called $4M$-groups. In the first part of this series of papers, we described simple $4M$-groups and nonsimple nonsolvable $4M$-groups without normal subgroups of prime index. In the second part, we started the investigation of finite nonsolvable $4M$-groups with a normal maximal subgroup using G. Pazderski's results on the structure of finite groups with exactly two conjugacy classes of maximal subgroups and the author's results on the structure of finite groups with exactly three conjugacy classes of maximal subgroups. The results of parts I and II are recalled in the introduction in Theorems 1–3. In the present third part, a complete description of finite nonsimple almost simple $4M$-groups is given (see Theorem 4). The proofs of the results are based on the works of many authors who studied the structure of maximal subgroups of finite simple and almost simple groups from various classes.

Keywords: finite group, almost simple group, maximal subgroup, $4M$-group.

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