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ASYMPTOTICS OF THE OPTIMAL TIME OF TRANSFERRING A LINEAR CONTROL SYSTEM WITH ZERO REAL PARTS OF THE EIGENVALUES OF THE MATRIX AT THE FAST VARIABLES TO AN UNBOUNDED TARGET SET

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This paper is devoted to a time-optimal control problem for a singularly perturbed linear autonomous system with smooth geometric constraints on the control and an unbounded target set:

$$\begin{cases} \dot{x} = y, & x,\, y \in \mathbb{R}^{2m}, \quad u \in \mathbb{R}^{2m}, \\ \varepsilon\dot{y} = Jy - Ju, & \|u\| \leqslant 1, \quad \varepsilon \ll 1, \\ x(0) = x^0, \quad y(0) = \varepsilon y^0, & \\ x(T_\varepsilon) = 0, \quad y(T_\varepsilon) \in \mathbb{R}^{2m}, \quad T_\varepsilon \longrightarrow \min, & \end{cases}$$

where

$$J = \begin{pmatrix} 0 & \beta \cdot I \\ -\beta \cdot I & 0 \end{pmatrix}, \quad \beta > 0.$$

The eigenvalues of the matrix J at the fast variables do not satisfy the standard requirement that the real part is negative. The solvability of the problem is proved. We also construct and justify a complete power asymptotic expansion in the sense of Erdelyi of the optimal time as the small parameter ε at the derivatives in the equations of the system tends to zero over some set. It is shown that the form of the asymptotics depends essentially on the set over which the small parameter tends to zero.

Keywords: optimal control, time-optimal control problem, asymptotic expansion, singularly perturbed problem, small parameter.

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