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2020 URAL WORKSHOP ON GROUP THEORY AND COMBINATORICS

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A review of the main events of the 2020 Ural Workshop on Group Theory and Combinatorics, held online on August 24–30, 2020, is presented, and a list of open problems with comments is given. Open problems were formulated by the participants at the Open Problems Session held at the end of the workshop.

Keywords: almost simple group, axial algebra of Jordan type, Cayley graph, chromatic number of a graph, conjugacy class sizes of a finite group, distance-regular graph, finite group, group with non-simple socle, Gruenberg–Kegel graph (prime garph), intersection of nilpotent subgroups, Moore graph, ordinary depth of a subgroup, partitions of a set, spectrum of a finite group, strongly regular graph, totally k-closed group, weak second maximal subgroup.

MSC: 17A99, 17C27, 20B15, 05C12, 05C15, 05C25, 05C50, 05C99, 05E18, 05E30, 20B05, 20B25, 20C15, 20D05, 20D06, 20D08, 20D15, 20D20, 20D30, 20D60, 20D99, 20E15, 20E32, 20E45, 20F29, 51E20

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