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## ON THE LATTICES OF THE $\omega$-FIBERED FORMATIONS OF FINITE GROUPS[1]

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Only finite groups and classes of finite groups are considered. The lattice approach to the study of formations of groups was first applied by A.N. Skiba in 1986. L.A. Shemetkov and A.N. Skiba established main properties of lattices of local formations and $\omega$-local formations where $\omega$ is a nonempty subset of the set $\mathbb{P}$ of all primes. An $\omega$-local formation is one of types of $\omega$-fibered formations introduced by V.A. Vedernikov and M.M. Sorokina in 1999. Let $f:\omega\cup\{\omega'\}\to\{\text{formations of groups}\}$, where $f(\omega')\neq\varnothing$, and $\delta:\mathbb{P}\to\{\text{nonempty Fitting formations}\}$ are the functions. Formation $\mathfrak{F}=(G\mid G/O_\omega(G)\in f(\omega')$ and $G/G_{\delta(p)}\in f(p)$ for all $p\in\omega\cap\pi(G))$ is called an $\omega$-fibered formation with a direction $\delta$ and with an $\omega$-satellite $f$, where $O_\omega(G)$ is the largest normal $\omega$-subgroup of the group $G$, $G_{\delta(p)}$ is the $\delta(p)$-radical of the group $G$, i.e. the largest normal subgroup of the group $G$ belonging to the class $\delta(p)$, and $\pi(G)$ is the set of all prime divisors of the order of the group $G$. We study properties of lattices of $\omega$-fibered formations of groups. In this work we have proved the modularity of the lattice $\Theta_{\omega\delta}$ of all $\omega$-fibered formations with the direction $\delta$. Its sublattice $\Theta_{\omega\delta}(\mathfrak{F})$ for the definite $\omega$-fibered formation $\mathfrak{F}$ with the direction $\delta$ is considered. We have established sufficient conditions under which the lattice $\Theta_{\omega\delta}(\mathfrak{F})$ is a distributive lattice with complements.

Keywords: finite group, class of groups, formation, $\omega$-fibered formation, lattice, modular lattice, distributive lattice, lattice with complements.

### REFERENCES

1. Birkhoff G. *Lattice Theory.* Providence, R.I.: Amer. Math. Soc., 1967, 418 p. ISBN: 0-8218-1025-1. Translated to Russian under the title *Teoriya reshetok.* Moscow: Nauka Publ., 1984, 568 p.
2. Doerk K., Hawkes T. *Finite soluble groups.* Berlin; New York: Walter de Gruyter, 1992, 901 p. ISBN: 3-11-012892-6.
3. Gaschütz W. Zur Theorie der endlichen auflösbaren Gruppen. *Math Z.*, 1962, vol. 80, no. 1, pp. 300–305. doi: 10.1007/BF01162386.
4. Hartley B. On Fischer's dualization of formation theory. *Proc. London Math. Soc.*, 1969, vol. 3, no. 9. pp. 193–207. doi: 10.1112/plms/s3-19.2.193.
5. Kamornikov S.F., Selkin M.V. *Subgroup functors and classes of finite groups.* Minsk: Belarusskaya Nauka Publ., 2003, 354 p. ISBN: 985-08-0556-0.
6. Kamozina O.V. Non-singly-generated multiply $\omega$-fibered Fitting classes of finite groups. *Math. Notes*, 2006, vol. 79, no. 3. pp. 366–376. doi: 10.4213/mzm2709.
7. Kamozina O.V. Algebraic lattices of multiply $\Omega$-foliated Fitting classes. *Discrete Math. Appl.*, 2006, vol. 16, no. 3, pp. 299–305. doi: 10.4213/dm53.
8. Shemetkov L.A. *Formatsii konechnykh grupp* [Formations of finite groups]. Moscow: Nauka Publ., 1978, 272 p.
9. Shemetkov L.A., Skiba A.N. *Formations of algebraic systems.* Moscow: Nauka Publ., 1989, 255 p. ISBN: 5-02-013918-1.
10. Skachkova (Yelovikova) Yu.A. Lattices of $\Omega$-fibered formations. *Discrete Math. Appl.*, 2002, vol. 12, no. 3, pp. 269–278. doi: 10.4213/dm243.
11. Skachkova (Yelovikova) Yu.A. Boolean lattices of multiply $\Omega$-foliated formations. *Discrete Math. Appl.*, 2002, vol. 12, no. 5, pp. 477–482. doi: 10.4213/dm252.

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12. Skiba A.N. On local formations of the length 5. In: *Arithmetic and subgroup structure of finite groups*. Minsk: Nauka i Tekhnika Publ., 1986, pp. 149–156.
13. Skiba A.N. *Algebra of formations*. Minsk: Belarusskay Nauka Publ., 1997, 240 p. ISBN: 985-08-0078-x .
14. Skiba A.N., Shemetkov L.A. Multiply $\mathfrak{L}$-composite formations of finite groups. *Ukr. Math. J.*, 2000, vol. 52, no. 6, pp. 898–913. doi: 10.1007/BF02591784 .
15. Skiba A.N., Shemetkov L.A. Multiply $\omega$-local formations and Fitting classes of finite groups. *Siberian Adv. Math.*, 2000, vol. 10, no. 2, pp. 112–141.
16. Vasil'ev A.F., Kamornikov S.F., Semenchuk V.N. On the lattices of subgroups of finite groups. In: *Infinite groups and adjacent algebraic systems*. Kiev: Institute of Mathematics of Academy of Sciences of Ukraine, 1993, pp. 27–54.
17. Vedernikov V.A., Sorokina M.M. $\Omega$-Foliated formations and Fitting classes of finite groups. *Discrete Math. Appl.*, 2001, vol. 11, no. 5, pp. 507–527. doi: 10.4213/dm299 .
18. Vedernikov V.A., Sorokina M.M. $\omega$-Fibered formations and Fitting classes of finite groups. *Math. Notes*, 2002, vol. 71, no. 1, pp. 39–55. doi: 10.4213/dm299 .
19. Vedernikov V.A. On new types of $\omega$-fibered formations of finite groups. In: *Ukrainian Mathematical Congress – 2001. Section 1.* Kiev: Inst. Matematiki NAN Ukrainy, 2002, ISBN: 966-02-2732-9 , pp. 36–45.
20. Vorob'ov N.N. *Algebra of classes of finite groups*. Vitebsk: BSU named after P.M. Masherov, 2012, 322 p. ISBN: 978-985-517-393-0 .
21. Wielandt H. Eine Verallgemeinerung der invarianten Untergruppen. *Math Z.*, 1939, vol. 45, pp. 209–244. doi: 10.1007/BF01580283 .

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