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ON THE CONVERGENCE OF MINIMIZERS AND MINIMUM VALUES IN VARIATIONAL PROBLEMS WITH POINTWISE FUNCTIONAL CONSTRAINTS IN VARIABLE DOMAINS

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We consider a sequence of convex integral functionals $F_s : W^{1,p}(\Omega_s) \to \mathbb{R}$ and a sequence of weakly lower semicontinuous and, in general, non-integral functionals $G_s : W^{1,p}(\Omega_s) \to \mathbb{R}$, where $\{\Omega_s\}$ is a sequence of domains in $\mathbb{R}^n$ contained in a bounded domain $\Omega \subset \mathbb{R}^n$ ($n \geqslant 2$) and $p > 1$. Along with this, we consider a sequence of closed convex sets $V_s = \{v \in W^{1,p}(\Omega_s) : M_s(v) \leqslant 0 \text{ a.e. in } \Omega_s\}$, where M_s is a mapping from $W^{1,p}(\Omega_s)$ to the set of all functions defined on Ω_s. We describe conditions under which minimizers and minimum values of the functionals $F_s + G_s$ on the sets V_s converge to a minimizer and the minimum value of a functional on the set $V = \{v \in W^{1,p}(\Omega) : M(v) \leqslant 0 \text{ a.e. in } \Omega\}$, where M is a mapping from $W^{1,p}(\Omega)$ to the set of all functions defined on Ω. In particular, for our convergence results, we require that the sequence of spaces $W^{1,p}(\Omega_s)$ is strongly connected with the space $W^{1,p}(\Omega)$ and the sequence $\{F_s\}$ Γ-converges to a functional defined on $W^{1,p}(\Omega)$. In so doing, we focus on the conditions on the mappings M_s and M which, along with the corresponding requirements on the involved domains and functionals, ensure the convergence of solutions of the considered variational problems. Such conditions have been obtained in our recent work, and, in the present paper, we advance in studying them.

Keywords: variational problem, integral functional, pointwise functional constraint, minimizer, minimum value, Γ-convergence, strong connectedness, variable domains.

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