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FINITE TOTALLY k-CLOSED GROUPS[1]

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For a positive integer k, a group G is said to be totally k-closed if in each of its faithful permutation representations, say on a set Ω, G is the largest subgroup of $\mathrm{Sym}(\Omega)$ which leaves invariant each of the G-orbits in the induced action on $\Omega \times \cdots \times \Omega = \Omega^k$. We prove that every finite abelian group G is totally $(n(G)+1)$-closed, but is not totally $n(G)$-closed, where $n(G)$ is the number of invariant factors in the invariant factor decomposition of G. In particular, we prove that for each $k \geq 2$ and each prime p, there are infinitely many finite abelian p-groups which are totally k-closed but not totally $(k-1)$-closed. This result in the special case $k = 2$ is due to Abdollahi and Arezoomand. We pose several open questions about total k-closure.

Keywords: permutation group, k-closure, totally k-closed group.

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