TRUDY INSTITUTA MATEMATIKI I MEKHANIKI UrO RAN

Vol. 27 No. 1 **2021**

A REMARK ON THE CONNECTION BETWEEN THE SECOND DIVIDED DIFFERENCE AND THE SECOND DERIVATIVE

Yu. S. Volkov

MSC: 41A15

DOI: 10.21538/0134-4889-2021-27-1-19-21

In the recent paper of S.I. Novikov and V.T. Shevaldin, the problem of the relationship between the second divided difference and the second derivative has been considered. The problem is to find the smallest value (in the uniform norm) of the second derivative among the functions interpolating a sequence of values with bounded second divided differences on arbitrary grids. In their paper, two-sided estimates for the required quantity have been found. We note that a more exact upper bound is known; it is attainable, for example, on a uniform grid. This bound can be easily obtained using Subbotin's interpolation splines.

Keywords: Favard problem, interpolation, divided difference, quadratic splines.

REFERENCES

1. Novikov S.I., Shevaldin V.T. On the connection between the second divided difference and the second derivative. *Tr. Inst. Mat. Mekh. UrO RAN*, 2020, vol. 26, no. 2, pp. 216–224 (in Russian). doi: 10.21538/0134-4889-2020-26-2-216-224 .
2. Subbotin Yu.N. On the relations between finite differences and the corresponding derivatives, *Amer. Math. Soc. Translations*, 1967, pp. 23–42.
3. Schoenberg I.J. Cardinal interpolation and spline functions. *J. Approx. Theory*, 1969, vol. 2, no. 2, pp. 167–206. doi: 10.1016/0021-9045(69)90040-9 .
4. Favard J. Sur l'interpolation. *J. Math. Pures Appl.*, 1940, vol. 19, no. 3, pp. 281–306.
5. de Boor C. How small can one make the derivatives of an interpolating function? *J. Approx. Theory*, 1975, vol. 13, no. 2, pp. 105–116. doi: 10.1016/0021-9045(75)90043-X .
6. de Boor C. A smooth and local interpolant with "small" k-th derivative, *Numerical solutions of boundary value problems for ordinary differential equations* (*Proc. Sympos., Univ. Maryland, Baltimore, Md., 1974*). N Y: Acad. Press, 1975, pp. 177–197.
7. Stechkin S.B., Subbotin Yu.N. *Splainy v vychislitel'noi matematike* [Splines in computational mathematics]. Moscow: Nauka Publ., 1976, 248 p.
8. Kantorovich L.V., Krylov V.I. *Approximate methods of higher analysis.* N Y: Interscience, 1964, 681 p. ISBN: 0471456721 .
9. de Boor C. On the (bi)infinite case of Shadrin's theorem concerning the L_∞-boundedness of the L_2-spline projector. *Proc. Steklov Inst. Math.*, 2012, vol. 277, suppl. 1, pp. 73–78. doi: 10.1134/S0081543812050082 .
10. Volkov Yu.S. Interpolation by splines of even degree according to Subbotin and Marsden. *Ukrainian Math. J.*, 2014, vol. 66, no. 7, pp. 994–1012. doi: 10.1007/s11253-014-0990-z .

Received October 23, 2020
Revised February 26, 2021
Accepted March 1, 2021

Yuriy Stepanovich Volkov, Dr. Phys.-Math. Sci., Prof., Sobolev Institute of Mathematics of the Siberian Branch of the Russian Academy of Sciences, Novosibirsk, 630090 Russia,
e-mail: volkov@math.nsc.ru .

Cite this article as: Yu. S. Volkov. A remark on the connection between the second divided difference and the second derivative, *Trudy Instituta Matematiki i Mekhaniki URO RAN*, 2021, vol. 27, no. 1, pp. 19–21 .