

**ASYMPTOTIC BEHAVIOR OF A SOLUTION FOR ONE CLASS
OF NONLINEAR INTEGRO-DIFFERENTIAL EQUATIONS
IN THE INCOME DISTRIBUTION PROBLEM**

A. Kh. Khachatryan, Kh. A. Khachatryan, H. S. Petrosyan

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We study a class of nonlinear integro-differential equations of convolution type, which have direct application in econometrics. Some qualitative properties of the solution are studied: its asymptotic behavior, monotonicity, and smoothness. A specific example of an applied nature is given.

Keywords: wealth distribution, asymptotics, wavefront, solution limit, monotonicity.

REFERENCES

1. Sargan I.D. The distribution of wealth. *Econometrics*, 1957, vol. 25, no. 4, pp. 568–590. doi: 10.2307/1905384.
2. Bellman R., Cooke K.L. *Differential-difference equations*. New York: Academic Press, 1963, 462 p. ISBN: 9780080955148. Translated to Russian under the title *Differentsial'no-raznostnye uravneniya*. Moscow: Mir Publ., 1967, 548 p.
3. Mondelbrot B. The Pareto–Lévy law and distribution of income. *Int. Economic Rev.*, 1960, vol. 1, no. 2, pp. 79–106. doi: 10.2307/2525289.
4. Champernowne D.G. A model of income distribution. *Economic J.*, 1953, vol. 63, no. 250, pp. 318–351. doi: 10.2307/2227127.
5. Chen Yu, Liao Yujie, Zhang Qi and Zhang Weiping. Ruin probabilities for the phase-type dual model perturbed by diffusion. *Communications in Statistics - Theory and Methods*, 2020, pp. 1–19. doi: 10.1080/03610926.2020.1737126.
6. Daliri Birjandi M.H., Saberi-Nadjafi J., Ghorbani A. An efficient numerical method for a class of nonlinear Volterra integro-differential equations. *J. Appl. Math.*, 2018, vol. 2018, art.-no. 7461058, 7 p. doi: 10.1155/2018/7461058.
7. Khachatryan A.Kh., Khachatryan Kh.A. On an integro-differential equation in the problem of wealth distribution. *Ekonomika i Mat. Metody TsEMI RAN*, 2009, vol. 45, no. 4, pp. 84–96.
8. Khachatryan A.Kh., Khachatryan Kh.A. On the solvability of a nonlinear integro-differential equation arising in the income distribution problem. *Comput. Math. and Math. Phys.*, 2010, vol. 50, no. 10, pp. 1702–1711. doi: 10.1134/S0965542510100064.
9. Khachatryan A., Khachatryan Kh. On solvability of a nonlinear problem in theory of income distribution. *Eurasian Math. J.*, 2011, vol. 2, no. 2, pp. 75–88.
10. Kolmogorov A.N., Fomin S.V. *Elements of the theory of functions and functional analysis*. Two volumes in one, translated from the first Russian edition 1957–1961. United States: Martino Fine Books, 2012, 280 p. ISBN: 1614273049. Original Russian text published in Kolmogorov A.N., Fomin S.V. *Elementy teorii funktsii i funktsional'nogo analiza*. Moscow: Nauka Publ., 1980.
11. Khachatryan Kh.A., Petrosyan H.S. On the solvability of a class of nonlinear Hammerstein–Stieltjes integral equations on the whole line. *Proc. Steklov Inst. Math.*, 2020, vol. 308, pp. 238–249. doi: 10.1134/S0081543820010198.
12. Diekmann O. Thresholds and travelling waves for the geographical spread of infection. *J. Math. Biology.*, 1978, vol. 6, no. 2, pp. 109–130. doi: 10.1007/BF02450783.

13. Diekmann O. Run for your life. A note on the asymptotic speed of propagation of an epidemic. *J. Diff. Eq.*, 1979, vol. 33, no. 1, pp. 58–73. doi: 10.1016/0022-0396(79)90080-9.
14. Diekmann O., Kaper H.G. On the bounded solutions of a nonlinear convolution equation. *Nonlinear Analysis: Theory, Methods & Applications*, 1978, vol. 2, no. 6, pp. 721–737. doi: 10.1016/0362-546X(78)90015-9.
15. Diekmann O., Gyllenberg M. Equations with infinite delay: Blending the abstract and the concrete. *J. Diff. Eq.*, 2012, vol. 252, no. 2, pp. 819–851. doi: 10.1016/j.jde.2011.09.038.
16. Budak B.M., Fomin S.V. *Multiple integrals, field theory and series. An advanced course in higher mathematics*. Moscow: Mir Publ., 1973, 640 p. ISBN: 0828520968. Original Russian text published in Budak B.M., Fomin S.V. *Kratnye integraly i ryady*. Moscow: Nauka Publ., 1966.
17. Gevorkyan G.G., Engibaryan N.B. New theorems for the renewal integral equation. *J. Contemp. Math. Anal., Armen. Acad. Sci.*, 1997, vol. 32, no. 1, pp. 2–16.
18. Engibaryan N.B. Renewal equations on the semi-axis. *Izv. Math.*, 1999, vol. 63, no. 1, pp. 57–71. doi: 10.1070/im1999v063n01ABEH000228.
19. Rudin W. *Functional Analysis*. New York: McGraw-Hill, 1973, 397 p. ISBN: 978-0070542259. Translated to Russian under the title *Funktsional'nyi analiz*. Moscow: Mir Publ., 1975, 444 p.
20. Pólya G., Szegő G. *Problems and theorems in analysis. Vol. 1*. Berlin: Springer, 1972, 392 p. doi: 10.1007/978-1-4757-1640-5. Translated to Russian under the title *Zadachi i teoremy iz analiza*. T. 1. Moscow: Nauka Publ., 1978, 392 p.

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Aghavard Khachaturovich Khachatryan, Dr. Phys.-Math. Sci., Armenian National Agrarian University, 0009, Yerevan, Republic of Armenia, e-mail: aghavard59@mail.ru.

Khachatur Aghavardovich Khachatryan, Dr. Phys.-Math. Sci., Institute of Mathematics, National Academy of Sciences of Armenia, 0019, Yerevan, Republic of Armenia; Lomonosov Moscow State University, Faculty of Mechanics and Mathematics, Moscow, 119991 Russia, e-mail: Khach82@rambler.ru.

Haykanush Samvelovna Petrosyan, Cand. Sci. (Phys.-Math.), Armenian National Agrarian University, 0009, Yerevan, Republic of Armenia; Lomonosov Moscow State University, Faculty of Mechanics and Mathematics, Moscow, 119991 Russia, e-mail: Haykuhi25@mail.ru.

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