

CRITERIA FOR THE EXISTENCE OF PERIODIC AND BOUNDED SOLUTIONS OF THREE-DIMENSIONAL SYSTEMS OF DIFFERENTIAL EQUATIONS**E. Mukhamadiev, A. N. Naimov**

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We study necessary and sufficient conditions for the existence of periodic and bounded solutions for one class of three-dimensional systems of nonlinear ordinary differential equations. The main nonlinear positively homogeneous terms are distinguished in the systems under consideration. Criteria for the existence of periodic and bounded solutions are formulated in terms of the properties of main nonlinear terms. For the periodic problem, it is proved that the previously known sufficient condition for the existence of periodic solutions is also a necessary condition. In the question of the existence of bounded solutions, it is proved that under a homotopy of the main nonlinear terms, the property of the existence of bounded solutions is preserved. Based on this, a new criterion for the existence of bounded solutions is proved with the use of methods of the qualitative theory of differential equations and nonlinear analysis, including the method of guiding functions and the topological method of Vazhevsky. The obtained results can be further generalized for multidimensional systems of differential equations.

Keywords: three-dimensional system of nonlinear ordinary differential equations, periodic solution, bounded solution, method of guiding functions, rotation of a vector field, homotopy relation, Vazhevsky topological method.

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