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ENDOMORPHISMS OF FINITE COMMUTATIVE GROUPOIDS RELATED WITH MULTILAYER FEEDFORWARD NEURAL NETWORKS

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In this paper, we introduce commutative, but generally not associative, groupoids $\mathrm{AGS}(\mathcal{N})$ consisting of idempotents. The groupoid $(\mathrm{AGS}(\mathcal{N}), +)$ is closely related to the multilayer feedforward neural networks $\mathcal{N}$ (hereinafter just a neural network). It turned out that in such neural networks, specifying a subnet of a fixed neural network is tantamount to specifying some special tuple composed of finite sets of neurons in the original network. All special tuples defining some subnet of the neural network $\mathcal{N}$ are contained in the set $\mathrm{AGS}(\mathcal{N})$. The rest of the tuples from $\mathrm{AGS}(\mathcal{N})$ also have a neural network interpretation. Thus, $\mathrm{AGS}(\mathcal{N}) = F_1 \cup F_2$, where F_1 is the set of tuples that induce subnets and F_2 is the set of other tuples. If two subnets of a neural network are specified, then two cases arise. In the first case, a new subnet can be obtained from these subnets by merging the sets of all neurons of these subnets. In the second case, such a merger is impossible due to neural network reasons. The operation $(+)$ for any tuples from $\mathrm{AGS}(\mathcal{N})$ returns a tuple that induces a subnet or returns a neutral element that does not induce subnets. In particular, if for two elements from F_1 the operation $(+)$ returns a neutral element, then the subnets induced by these elements cannot be combined into one subnet. For any two elements from $\mathrm{AGS}(\mathcal{N})$, the operation has a neural network interpretation. In this paper, we study the algebraic properties of the groupoids $\mathrm{AGS}(\mathcal{N})$ and construct some classes of endomorphisms of such groupoids. It is shown that every subnet $\mathcal{N}'$ of the net $\mathcal{N}$ defines a subgroupoid T in the groupoid $\mathrm{AGS}(\mathcal{N})$ isomorphic to $\mathrm{AGS}(\mathcal{N}')$. It is proved that for every finite monoid G there is a neural network $\mathcal{N}$ such that G is isomorphically embeddable into the monoid of all endomorphisms $\mathrm{End}(\mathrm{AGS}(\mathcal{N}))$. This statement is the main result of the work.

Keywords: groupoid endomorphism, multilayer feedforward neural networks, multilayer neural network subnet.

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