TRUDY INSTITUTA MATEMATIKI I MEKHANIKI UrO RAN

Vol. 27 No. 1 **2021**

UDK 512.542, 512.547

ON A QUESTION CONCERNING THE TENSOR PRODUCT OF MODULES

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MSC: 20C33, 20B15, 20C20, 20D06

DOI: 10.21538/0134-4889-2021-27-1-103-109

Assume that G is a group, K is an algebraically closed field, and V_1 and V_2 are KG-modules. The following question is considered: under what constraints on G, K, V_1, and V_2 does $V_1 \otimes V_2 \cong V_1 \otimes I$ hold, where I is the trivial KG-module (of dimension $\dim(V_2)$)? Earlier, when considering a problem of P. Cameron on finite primitive permutation groups, the author obtained and used some results on this question. This work continues the study of the question. The following results were obtained. 1. Assume that G is a nontrivial connected reductive algebraic group, and V_1 and V_2 are faithful semisimple KG-modules. Then $V_1 \otimes V_2 \ncong V_1 \otimes I$. 2. Assume that G is a nontrivial finite group, $\mathrm{char}(K) = 0$, V_1 is a KG-module, and V_2 is a faithful KG-module. Then $V_1 \otimes V_2 \cong V_1 \otimes I$ if and only if V_1 is the direct sum of $\frac{\dim(V_1)}{|G|}$ regular KG-modules. In addition, we consider the question of the possibility that $V_1 \otimes V_2 \cong V_1 \otimes I$ in the case where $G = SL_2(p^n)$, V_1 and V_2 are simple KG-modules, and $\mathrm{char}(K) = p$.

Keywords: finite group, algebraic group, group representation, tensor product of modules.

REFERENCES

1. Cameron P.J. Suborbits in transitive permutation groups. In: Hall M., van Lint J.H. (eds), *Combinatorics.* NATO Advanced Study Institutes Series, vol. 16. Dordrecht: Springer, 1974, pp. 419–450. doi: 10.1007/978-94-010-1826-5_20 .
2. Curtis C.W., Reiner I. *Representation theory of finite groups and associative algebras.* Pure and Applied Mathematics, vol. XI. N Y; London: Interscience Publishers, 1962, 689 p. ISBN: 978-0-8218-4066-5 .
3. Doty S., Henke A. Decomposition of tensor products of modular irreducibles for SL_2. *Q. J. Math.*, 2005, vol. 56, no. 2, pp. 189–207. doi: 10.1093/qmath/hah027 .
4. Fulton W., Harris J. *Representation theory: A first course.* Graduate Texts in Mathematics, Readings in Mathematics, vol. 129. New York: Springer, 2004, 551 p. doi: 10.1007/978-1-4612-0979-9 .
5. Jantzen J.C. *Representations of algebraic groups.* Mathematical Surveys and Monographs, vol. 107. Providence: American Mathematical Society, 2003, 576 p. ISBN: 978-0-8218-4377-2 .
6. Humphreys J.E. *Introduction to Lie algebras and representation theory.* New York: Springer, 1980, 173 p. ISBN: 0-387-90052-7/,.
7. Humphreys J.E. *Modular representations of finite groups of Lie type.* Cambridge: Cambridge University Press, 2011, 206 p. doi: 10.1017/CBO9780511525940 .
8. Liebeck M.W., Nikolov N., Shalev A. Groups of Lie type as products of SL_2 subgroups. *J. Algebra*, 2011, vol. 326, no. 1, pp. 201–207. doi: 10.1016/j.jalgebra.2008.12.030 .
9. Malle G., Testerman D. *Linear algebraic groups and finite groups of Lie type.* Cambridge: Cambridge Univ. Press, 2011, 309 p. doi: 10.1017/CBO9780511994777 .

Received November 22, 2020
Revised December 30, 2020
Accepted January 11, 2021

Funding Agency: This work was supported by the Russian Foundation for Basic Research (project no. 20-01-00456).

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Cite this article as: A. V. Konygin. On a question concerning the tensor product of modules, *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2021, vol. 27, no. 1, pp. 103–109 .