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DISTANCE-REGULAR GRAPHS WITH INTERSECTION ARRAYS $\{104, 70, 25; 1, 7, 80\}$ AND $\{272, 210, 49; 1, 15, 224\}$ DO NOT EXIST

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I. N. Belousov, A. A. Makhnev, and M. S. Nirova in 2019 described Q-polynomial distance-regular graphs Γ of diameter 3 with strongly regular graphs Γ_2 and Γ_3, where the graphs Γ_2 and Γ_3 have the same vertices as Γ and these vertices are adjacent if and only if they are at distance 2 and 3 in Γ, respectively. Some of the Q-polynomial distance-regular graphs Γ with strongly regular graphs Γ_2 and Γ_3 have intersection arrays

$$\left\{\frac{(s^2+su-1)(u^2-1)}{s^2-1}, \frac{(u^2-s^2)su}{s^2-1}, u^2; 1, \frac{u^2-s^2}{s^2-1}, \frac{su^3-su}{s^2-1}\right\}.$$

For small values of s and u, we have intersection arrays $\{104, 70, 25; 1, 7, 80\}$ ($u = 5$, $s = 2$) and $\{272, 210, 49; 1, 15, 224\}$ ($u = 7$, $s = 2$). We prove that distance-regular graphs with such arrays do not exist. We also study the properties of a local subgraph in a hypothetical distance-regular graph with intersection array $\{399, 320, 64; 1, 20, 336\}$ ($u = 8$, $s = 2$).

Keywords: distance-regular graph, Q-polynomial graph.

MSC: 05C25

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