

EULER POLYNOMIALS IN THE PROBLEM OF EXTREMAL FUNCTIONAL INTERPOLATION IN THE MEAN

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The problem of extremal functional interpolation in the mean, first studied by Yu. N. Subbotin, is considered. Representations of the extremal functions solving this problem in terms of Euler polynomials are found, and their properties are studied. This made it possible to calculate the values of the extremal interpolation constants in terms of easily computable values of the Euler polynomials at certain points and sometimes Favard constants. The compatibility of the constants of extremal functional interpolation in the mean as the value of the averaging interval tends to zero with the constants of extremal functional interpolation is demonstrated.

Keywords: Euler polynomials, Favard constants, interpolation in the mean.

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