

**UPPER ESTIMATES FOR BEST MEAN-SQUARE APPROXIMATIONS
FOR SOME CLASSES OF BIVARIATE FUNCTIONS
BY FOURIER–CHEBYSHEV SUMS**

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In space $L_{2,\rho}$ of bivariate functions summable with square on set $Q = [-1, 1]^2$ with weight $\rho(x, y) = 1/\sqrt{(1-x^2)(1-y^2)}$ the sharp inequalities of Jackson–Stechkin type in which the best polynomial approximation estimated above by Peetre \mathcal{K} -functional were obtained. We also find the exact values of various widths of classes of functions defined by generalized modulus of continuity and \mathcal{K} -functionals. Also the exact upper bounds for modules of coefficients of Fourier–Tchebychev on considered classes of functions were calculated.

Keywords: mean-squared approximation, generalized modulus of continuity, Fourier–Tchebychev double series, translated operator.

MSC: 42A10, 41A17, 41A44

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