PERIODIC WAVELETS ON A MULTIDIMENSIONAL SPHERE AND THEIR APPLICATION FOR FUNCTION APPROXIMATION

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The author's scheme for constructing a multiresolution analysis on a sphere in $\mathbb{R}^3$ with respect to the spherical coordinates, which was published in 2019, is extended to spheres in $\mathbb{R}^n$ ($n \geq 3$). In contrast to other papers, only periodic wavelets on the axis and their tensor products are used. Approximation properties are studied only for the wavelets based on the simplest scalar wavelets of Kotel'nikov–Meyer type with the compact support of their Fourier transforms. The implementation of the idea of a smooth continuation of functions from a sphere to 2π-periodic functions in the polar coordinates analytically (without the complicated geometric interpretation made by the author earlier in $\mathbb{R}^3$) turned out to be very simple.

Keywords: wavelet, scaling function, approximation.

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