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GEOMETRIC APPROACH TO FINDING CONSTRAINED EXTREMA

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In this paper, we give a geometric interpretation and a geometric proof of the necessary condition for the existence of a constrained extremum. The presented approach can be applied to finding constrained extrema of nondifferentiable functions (i.e., when Lagrange's method of undetermined multipliers is not applicable in the "classical" form). The following examples are considered: the inequality of arithmetic and geometric means, Young's inequality for products, and Jensen's inequality.

Keywords: constrained extremum, level surface, Lagrange multipliers.

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