

INTERPOLATING ORTHOGONAL BASES OF AN MRA AND WAVELETS

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The main goal of this paper is to construct orthonormal bases of a multiresolution analysis (MRA) that are interpolating on the grid $k/2^j$. We consider an orthonormal MRA and the corresponding wavelets. Based on this MRA and using orthogonal masks of the scaling functions, we construct new masks of scaling functions that satisfy the interpolation condition. In I. Daubechies's book it is proved that bases of an MRA that are interpolating and orthogonal simultaneously cannot have a compact support. In 2008, Yu.N. Subbotin and N.I. Chernykh suggested a method for modifying the Meyer scaling function in such a way that the basis formed by it is simultaneously orthogonal and interpolating. In the present paper we propose a method for modifying a wider class of scaling functions in such a way that the new scaling functions remain orthogonal and at the same time become interpolating. We start the construction with a mask of a scaling function and find necessary and sufficient conditions for the shifts of the scaling function obtained with the use of the modified mask to form an interpolating orthogonal system.

Keywords: orthogonal wavelet, interpolating wavelet, scaling function, basis, multiresolution analysis, mask of scaling function.

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