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EXTREMAL INTERPOLATION ON THE SEMIAXIS WITH THE SMALLEST NORM OF THE THIRD DERIVATIVE

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The following problem is considered. For a class of interpolated sequences $y = \{y_k\}_{k=-\infty}^{+\infty}$ of real numbers such that their third-order divided difference constructed for arbitrary knots $\{x_k\}_{k=-\infty}^{+\infty}$ are bounded in absolute value by a fixed positive number, it is required to find a function f having the third derivative almost everywhere and such that $f(x_k) = y_k$ $(k \in \mathbb{Z})$ and the third derivative has the smallest L_∞-norm. The problem is solved on the positive semiaxis $\mathbb{R}_+ = (0, +\infty)$ for geometric grids in which the sequence of steps $h_k = x_{k+1} - x_k$ $(k \in \mathbb{Z})$ is a geometric progression with ratio p $(p > 1)$; i.e., $h_{k+1}/h_k = p$. In the case of a uniform grid $x_k = kh$ $(h > 0, k \in \mathbb{Z})$ on the whole axis $\mathbb{R}$ (i.e., for $p = 1$), this problem was solved by Yu. N. Subbotin in 1965 and is known as the Yanenko–Stechkin–Subbotin problem of extremal function interpolation.

Keywords: interpolation, divided difference, splines, difference equation.

MSC: 41A15

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