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ON A REFINEMENT OF MARCINKIEWICZ–ZYGMUND TYPE INEQUALITIES

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The main goal of this paper is to verify a refined Marcinkiewicz–Zygmund type inequality with a quadratic error term

$$\frac{1}{2}\sum_{j=0}^{nm-1}(x_{j+1}-x_{j-1})w(x_j)|t_n(x_j)|^q=(1+O(m^{-2}))\int_{-\pi}^{\pi}w(x)|t_n(x)|^q\,dx,\quad 2\le q<\infty,$$

where t_n is any trigonometric polynomial of degree at most n, $-\pi=x_0<x_1<\cdots<x_{mn}=\pi$, $\max\limits_{0\le j\le mn-1}(x_{j+1}-x_j)=O\Big(\dfrac{1}{nm}\Big)$, $m,n\in\mathbb{N}$, and w is a Jacobi type weight. Moreover, the quadratic error term $O(m^{-2})$ is shown to be sharp, in general. In addition, similar results are given for $q=\infty$ and in the multivariate case.

Keywords: multivariate polynomials, Marcinkiewicz–Zygmund, Bernstein, and Schur type inequalities, discretization of L^p norm, doubling and Jacobi type weights.

MSC: 41A17, 41A63

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