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MINIMAX SOLUTIONS OF HOMOGENEOUS HAMILTON–JACOBI EQUATIONS WITH FRACTIONAL-ORDER COINVARIANT DERIVATIVES

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The Cauchy problem is considered for a homogeneous Hamilton–Jacobi equation with fractional-order coinvariant derivatives, which arises in problems of dynamical optimization of systems described by differential equations with Caputo fractional derivatives. A generalized solution of the problem in the minimax sense is defined. It is proved that such a solution exists, is unique, depends continuously on the parameters of the problem, and is consistent with the classical solution. An infinitesimal criterion of the minimax solution is obtained in the form of a pair of differential inequalities for suitable directional derivatives. An illustrative example is given.

Keywords: Hamilton–Jacobi equations, generalized solutions, coinvariant derivatives, fractional-order derivatives.

MSC: 35F21, 34A08, 26A33

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