NONASSOCIATIVE ENVELOPING ALGEBRAS OF CHEVALLEY ALGEBRAS

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An algebra R is said to be an *exact enveloping algebra* for a Lie algebra L if L is isomorphic to the algebra $R^{(-)}$ obtained by replacing the multiplication in R by the commutation: $a * b := ab - ba$. We study exact enveloping algebras of certain subalgebras of a Chevalley algebra over a field K associated with an indecomposable root system Φ. The structure constants of the Chevalley basis of this algebra are chosen with a certain arbitrariness for the *niltriangular* subalgebra $N\Phi(K)$ with the basis $\{e_r \mid r \in \Phi^+\}$. The exact enveloping algebras R for $N\Phi(K)$, which were found in 2018, depend on this choice. The notion of standard enveloping algebra is introduced. For the type A_{n-1}, one of the exact enveloping algebras R is the algebra $NT(n, K)$ of all niltriangular $n \times n$ matrices over K. The theorem of R. Dubish and S. Perlis on the ideals of $NT(n, K)$ states that R is standard in this case. We prove that an associative exact enveloping algebra R of a Lie algebra $NT(n, K)$ of type A_{n-1} ($n > 3$) is unique and isomorphic to $NT(n, K)$ up to passing to the opposite algebra $R^{(\mathrm{op})}$. Standard enveloping algebras R are described. The existence of a standard enveloping algebra is proved for the Lie algebras $N\Phi(K)$ of all types excepting D_n ($n \geq 4$) and E_n ($n = 6, 7, 8$).

Keywords: Lie algebra, exact enveloping algebra, Chevalley algebra, niltriangular subalgebra, standard ideal.

REFERENCES

1. Kaplansky I. *Lie algebras and locally compact groups*, Chicago and London, The University of Chicago Press, 1971, 152 p. Translated to Russian under the title *Algebry Li i lokal'no kompaktnye gruppy*, Moscow: Mir Publ., 152 p.

2. Levchuk V.M. Niltriangular subalgebra of Chevalley algebra: the enveloping algebra, ideals and automorphisms. *Dokl. Math.*, 2018, vol. 97, no. 1, pp. 23–27. doi: 10.1134/S1064562418010088 .

3. Chevalley C. Sur certain groups simples. *Tôhoku Math. J.*, 1955, vol. 7, no. 1-2, pp. 14–66. doi: 10.2748/tmj/1178245104 .

4. Carter R. *Simple groups of Lie type*. N Y: Wiley and Sons, 1972, 331 p. ISBN: 0471137359 .

5. Dubish R., Perlis S. On total nilpotent algebras. *Amer. J. Math.*, 1951, vol. 73, no. 2, pp. 439–452. doi: 10.2307/2372186 .

6. Egorychev G.P., Levchuk V.M. Enumeration in the Chevalley algebras. *ACM SIGSAM Bulletin*, 2001, vol. 35, no. 2, pp. 20–34.

7. Levchuk V.M., Suleimanova G.S. Extremal and maximal normal abelian subgroups of a maximal unipotent subgroup in groups of Lie type. *J. Algebra*, 2012, vol. 349, no. 1, pp. 98–116. doi: 10.1016/j.algebra.2011.10.025 .

8. Serre J-P. *Lie algebras and Lie groups*. N Y; Amsterdam: Benjamin, 1965, 376 p. Translated to Russian under the title *Algebry Li i gruppy Li*. Moscow: Mir Publ., 1969, 376 p.

9. Levchuk V.M. Automorphisms of unipotent subgroups of Chevalley groups. *Algebra and Logic*, 1990, vol.29, no. 3, pp. 211–224. doi: 10.1007/BF01979936 .

10. Hodyunya N.D. Enumerations of ideals in niltriangular subalgebra of Chevalley algebras. *J. SFU Math. and Phys*, 2018, vol. 11, no. 3, pp. 271–277. doi: 10.17516/1997-1397-2018-11-3-271-277 .

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