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REFINEMENT OF ESTIMATES FOR THE LYAPUNOV EXPONENTS OF A CLASS OF LINEAR NONAUTONOMOUS SYSTEMS OF DIFFERENCE EQUATIONS

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We obtain an estimate for the norm of an nth-order square matrix A^t:

$$\|A^t\| \le \sum_{k=0}^{n-1} C_t^k \gamma^{t-k} (\gamma + \|A\|)^k, \quad t \ge n-1,$$

where C_t^k are the binomial coefficients, $\gamma = \max\limits_i |\lambda_i|$, and λ_i are the eigenvalues of A. Based on this estimate and using the freezing method, we improve the constants in the upper and lower estimates for the highest and lowest exponents, respectively, of the system $x(t+1) = A(t)x(t)$, $x \in \mathbb{R}^n$, $t \in \mathbb{Z}^+$, with a completely bounded matrix $A(t)$. It is assumed that the matrices $A(t)$ and $A^{-1}(t)$ satisfy the inequalities $\|A(t) - A(s)\| \le \delta|t-s|^\alpha$, $\|A^{-1}(t) - A^{-1}(s)\| \le \delta|t-s|^\alpha$ with some constants $0 < \alpha \le 1$ and $\delta > 0$ for any $t, s \in \mathbb{Z}^+$. We give an example showing that the constants γ and δ are generally related.

Keywords: estimates for Lyapunov exponents, freezing method for discrete systems.

MSC: 39A30, 39A22

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