TRUDY INSTITUTA MATEMATIKI I MEKHANIKI UrO RAN

Vol. 26 No. 3 **2020**

AUTOMORPHISMS OF RINGS OF NONFINITARY NILTRIANGULAR MATRICES

Yu. V. Bekker, D. V. Levchuk, E. A. Sotnikova

Let K be an associative ring with identity, and let Γ be an arbitrary linearly ordered set (briefly, chain). Matrices $\alpha = \|a_{ij}\|$ over K with indices i and j from Γ with respect to linear operations always form a K-module $M(\Gamma, K)$. The matrix multiplication in $M(\Gamma, K)$ is generally not defined if Γ is an infinite chain. The finitary matrices in $M(\Gamma, K)$ form a known ring with matrix multiplication and addition. On the other hand, as proved in 2019, for the chain $\Gamma = \mathbb{N}$ of natural numbers, the submodule in $M(\Gamma, K)$ of all (lower) niltriangular matrices with matrix multiplication and addition gives a radical ring $NT(\Gamma, K)$. Its adjoint group is isomorphic to the limit unitriangular group. The automorphisms of the group $UT(\infty, K)$ over a field K of order greater than 2 were studied by R. Slowik. In the present paper, it is proved that any infinite chain Γ is isometric or anti-isometric to the chain $\mathbb{N}$ or the chain of all integers if $NT(\Gamma, K)$ with matrix multiplication is a ring. When the ring of coefficients K has no divisors of zero, the main theorem shows that the automorphisms of $NT(\mathbb{N}, K)$ and of the associated Lie ring, as well as of the adjoint group, are standard.

Keywords: radical ring, Chevalley algebra, niltriangular subalgebra, unitriangular group, nonfinitary generalizations, automorphism.

MSC: 22E05

DOI: 10.21538/0134-4889-2020-26-3-7-13

REFERENCES

1. Levchuk V.M. Some locally nilpotent rings and their adjoined groups. *Mat. Zametki*, 1987, vol. 42, no. 5, pp. 848–853. doi: 10.1007/BF01137426 .
2. Kuzucuoglu F., Levchuk V.M. Isomorphism of certain locally nilpotent finitary groups and associated rings. *Acta Appl. Math.*, 2004, vol. 82, no. 2, pp. 169–181. doi: 10.1023/B:ACAP.0000027533.59937.14 .
3. Merzlyakov Yu.I. Equisubgroups of unitriangular groups: a criterion for self-normalizability. *Dokl. Math.*, 1995, vol. 50, no. 3, pp. 507–511.
4. Holubovski W. *Algebraic properties of groups of infinite matrices*. Abstract of Dr. Phys.–Math. Sci. Dissertation. St. Petersburg: St. Petersburg State Univ., 2007, 27 p.
5. Holubowski W. *Algebraicheskie svoistva grupp beskonechnykh matrits* [Algebraic properties of groups of infinite matrices]. Gliwice: Wydawnictwo Politechniki Slaskiej, 2017, 140 p.
6. Slowik R. Bijective maps of infinite triangular and unitriangular matrices preserving commutators. *Linear and Multilinear Algebra*, 2013, vol. 61, no. 8, pp. 1028–1040. doi: 10.1080/03081087.2012.728214 .
7. Levchuk V.M., Radchenko O.V. Derivations of the locally nilpotent matrix rings. *J. Algebra Appl.*, 2010, vol. 9, no. 5, pp. 717–724. doi: 10.1142/S0219498810004154 .
8. Holubowski W., Slowik R. Parabolic subgroups of groups of column-finite infinite matrices. *Linear Algebra Appl.*, 2012, vol. 437, no. 2, pp. 519–524. doi: 10.1016/j.laa.2012.02.020 .
9. Bekker J.V., Levchuk V.M., Sotnikova E.A. Non-finitary generalizations of nil-triangular subalgebras of Chevalley algebras. *The Bulletin of Irkutsk State University*, Series Mathematics, 2019, vol. 29, pp. 39–51 (in Russian). doi: 10.26516/1997-7670.2019.29.39 .
10. Kurosh A.G. *Obshchaya algebra* [General Algebra]. St. Petersburg: Lan' Publ, 1975, 262 p.
11. Kuratowski K., Mostowski A. *Set theory*. Studies in logic and the foundations of mathematics, vol. 53. Amsterdam: North-Holland, 1968, 417 p. ISBN: 9780444534170 .
12. Dubish R., Perlis S. On total nilpotent algebras. *Amer. J. Math.*, 1951, vol. 73, no. 3, pp. 439–452. doi: 10.2307/2372186 .
13. Levchuk V.M. Connections between a unitriangular group and certain rings. II. Groups of automorphisms. *Sib. Math. J.*, 1983, vol. 24, no. 4, pp. 543–557. doi: 10.1007/BF00969552 .

Received July 11, 2020
Revised July 22, 2020
Accepted August 10, 2020

Funding Agency: This work is supported by the Krasnoyarsk Mathematical Center, which is financed by the Ministry of Science and Higher Education of the Russian Federation within the project for the establishment and development of regional centers for mathematical research and education (agreement no. 075-02-2020-1534/1).

Julianna Vladimirovna Bekker, doctoral student, Siberian Federal University, Krasnoyarsk, 660041 Russia, e-mail: angel220@bk.ru .

Denis Vladimirovich Levchuk , Cand. Sci. (Phys.-Math.), Siberian Federal University, Krasnoyarsk, 660041 Russia, e-mail: Dlevchuk82@mail.ru .

Elena Andreevna Sotnikova, graduate student, Siberian Federal University, Krasnoyarsk, 660041, Russia, e-mail: olgaRV520@yandex.ru .

Yu. V. Bekker, D. V. Levchuk, E. A. Sotnikova. Automorphisms of rings of nonfinitary niltriangular matrices, *Trudy Instituta Matematiki i Mekhaniki URO RAN*, 2020, vol. 26, no. 3, pp. 7–13 .