

ON PRONORMALITY OF SECOND MAXIMAL SUBGROUPS IN FINITE GROUPS WITH SOCLE $L_2(q)$

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According to Ph. Hall, a subgroup H of a finite group G is called pronormal in G if, for any element g of G , the subgroups H and H^g are conjugate in $\langle H, H^g \rangle$. The simplest examples of pronormal subgroups of finite groups are normal subgroups, maximal subgroups, and Sylow subgroups. Pronormal subgroups of finite groups were studied by a number of authors. For example, Legovini (1981) studied finite groups in which every subgroup is subnormal or pronormal. Later, Li and Zhang (2013) described the structure of a finite group G in which, for a second maximal subgroup H , its index in $\langle H, H^g \rangle$ does not contain squares for any g from G . A number of papers by Kondrat'ev, Maslova, Revin, and Vdovin (2012–2019) are devoted to studying the pronormality of subgroups in a finite simple nonabelian group and, in particular, the existence of a nonpronormal subgroup of odd index in a finite simple nonabelian group. In the *Kourovka Notebook*, the author formulated Question 19.109 on the equivalence in a finite simple nonabelian group of the condition of pronormality of its second maximal subgroups and the condition of Hallness of its maximal subgroups. Tyutyaynov gave a counterexample $L_2(2^{11})$ to this question. In the present paper, we provide necessary and sufficient conditions for the pronormality of the second maximal subgroups in the group $L_2(q)$. In addition, for $q \leq 11$, we find the finite almost simple groups with socle $L_2(q)$ in which all second maximal subgroups are pronormal.

Keywords: finite group, simple group, maximal subgroup, pronormal subgroup.

MSC: 20D06, 20D30, 20E28

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REFERENCES

1. Legovini P. Gruppi finiti i cui sottogruppi sono o subnormali o pronormali. *Rend. Sem. Math. Univ. Padova*, 1981, vol. 65, pp. 47–51.
2. Li X., Zhang X. On indices of subgroups in the join of their conjugate pairs. *Siberian Math. J.*, 2013, vol. 54, no. 4, pp. 656–665. doi: 10.1134/S0037446613040071.
3. Vdovin E.P., Revin D.O. Pronormality of Hall subgroups in finite simple groups. *Siberian Math. J.*, 2012, vol. 53, no 3, pp. 419–430. doi: 10.1134/S0037446612020231.
4. Kondrat'ev A.S., Maslova N.V., Revin D.O. On the pronormality of subgroups of odd index in finite simple groups. *Siberian Math. J.*, 2015, vol. 56, no. 6, pp. 1101–1107. doi: 10.1134/S0037446615060142.
5. Kondrat'ev A.S., Maslova N.V., Revin D.O. A pronormality criterion for supplements to abelian normal subgroups. *Proc. Steklov Inst. Math.*, 2017, vol. 296, suppl. 1, pp. 145–150. doi: 10.1134/S0081543817020134.
6. *Unsolved problems in group theory. The Kourovka notebook* [e-resource], No. 19, eds. Evgeny Khukhro and Victor Mazurov, Novosibirsk, 2018 (2020), 250 p. Available at: <http://kourovka-notebook.org>.
7. Dickson L.E. *Linear Group: with an exposition of the Galois field theory*. Teubner: Leipzig, 1901. (N Y: Dover Publ., Inc. 1958).
8. Huppert B. *Endliche Gruppen I*. Berlin; Heidefferg; N Y: Springer-Verlag, 1967. 793 p.
9. Giudici M. *Maximal subgroups of almost simple groups with socle $PSL(2, q)$* . 2007. 11 p. Available on <https://arxiv.org/abs/math/0703685>.
10. Bray J.N. et al. *The maximal subgroups of the low-dimensional finite classical groups*. Cambridge: University Press, 2013, 435 p. doi: 10.1017/CBO9781139192576.
11. Conway J.H. et al. *Atlas of finite group*. Oxford: Clarendon Press, 1985, 252 p. ISBN: 0-19-853199-0.
12. Aschbacher M. *Finite group theory*. Cambridge: Cambridge University Press, 1986. 274 p.

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