

**OPEN QUESTIONS FORMULATED AT THE 13th SCHOOL–CONFERENCE
ON GROUP THEORY DEDICATED TO V. A. BELONOGOV’S 85th BIRTHDAY****N. V. Maslova, I. N. Belousov, N. A. Minigulov**

A review of the main events of the 13th School–Conference on Group Theory, which was held online on August 3–7, 2020, is presented, and a list of open questions with comments is given. Open questions were formulated by the participants at the Open Problems Session held at the end of the school–conference. Among the posed problems there are a series of questions on the characterization of a finite group by its arithmetic invariants such as the spectrum, the Gruenberg–Kegel graph, the solvable graph, and the degrees of irreducible complex characters (L. S. Kazarin, A. S. Kondrat’ev, and N. V. Maslova); the question of the conjugacy of the Sylow 2-subgroups in locally finite groups with additional conditions on these subgroups (V. D. Mazurov); a series of problems on the characterization of distance-regular graphs by their intersection arrays (A. A. Makhnev); the question of the nilpotent length of a finite solvable group whose Carter subgroup coincides with the Gaschütz subgroup (V. S. Monakhov); a series of problems about the structure of conjugately biprimively finite groups or Shunkov groups (A. I. Sozutov); a question on the structure of some matrix groups over a residue ring \mathbb{Z}_n for a positive integer n (V. A. Roman’kov); a question on the characterization of Mazurov triples in finite groups (A. V. Timofeenko); and other open questions of the modern group theory and its applications. V. A. Belonogov’s brief biography and the list of his main publications are also presented.

Keywords: finite group, spectrum, Gruenberg–Kegel graph, solvable graph, locally finite group, periodic group, Sylow 2-subgroup, Carter subgroup, Gaschütz subgroup, conjugately biprimively finite group (Shunkov group), Mazurov triple.

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