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DUALITY IN LINEAR ECONOMIC MODELS OF EXCHANGE

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A further development of an original approach to the equilibrium problem in a linear exchange model and its variations is presented. The conceptual basis of the approach is polyhedral complementarity. The original problem is reduced to a fixed point problem for a piecewise constant point-to-set mapping on the price simplex. For the model with fixed budgets (Fisher model), the emerging mapping is potential, and this provides a new reduction of the equilibrium problem to a pair of optimization problems. The problems are in duality similarly to linear programming problems. This reduction of the Fisher model differs from the well-known reduction of E. Eisenberg and D. Gale and allows a development of two finite algorithms for searching equilibrium prices. In this paper we present a new conceptually complete version of the approach. We give an explicit formulation of the dual variant of the obtained reduction for the Fisher model and its generalizations. The reduction of the equilibrium problem to an optimization problem is also obtained for the general exchange model with variable budgets.

Keywords: exchange model, economic equilibrium, fixed point, polyhedral complementarity, optimization problem, conjugate function, algorithm.

MSC: 90C33, 90C46

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