DISCRETE ORTHOGONAL TRANSFORMS ON MULTISETS ASSOCIATED WITH COMPLETE SEQUENCES

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We consider a specific version of the authors' approach to the synthesis of bases of discrete orthogonal transforms (DOTs). The approach takes into account the relation between the structure of basis functions of a transform and the existence of a certain numeral system on the (multidimensional) index set of the input signal. In contrast to Chernov's prototype paper "Discrete orthogonal transforms with bases generated by self-similar sequences" (2018), which was concerned with DOTs associated with irredundant numeral systems (where each index of the input signal has a unique representation in a chosen numeral system), in the present paper we study the case of the so-called complete numeral systems. In this case, there is no bijection between the set of input indices of DOTs and the set of their digital representations. Potentially, such statements of applied problems naturally appear in image recognition, artificial intelligence, theory of formal languages, mathematical programming, and other areas where the analyzed objects are characterized by many heterogeneous attributes, which can be quantitative, qualitative, and mixed. There may be several copies of each objects, and the copies may have inconsistent descriptions, which must be considered and analyzed as a whole. Such objects with many attributes can be represented as multisets ("sets with repetitions"). Since discrete spectral analysis is a basic tool for solving the described problems in the classical "multiple" interpretation of objects, we try to extend some ideas and methods of spectral analysis to the case of multiset objects.

Keywords: multisets, discrete orthogonal transformations, complete sequences.

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